4.3 Expected Values of Continuous Random Variables

Defn: The expected value of a continuous r.v. Y is

if $E(Y)$ exists, which is the case if

- This is analogous to $E(Y)$ for a discrete r.v., with an integral replacing a sum.

Note: If the support of Y is some subset of the real line, we need only to integrate over the support of Y to find $E(Y)$.

Theorem: If $g(Y)$ is a function of a continuous r.v. Y, then

if the integral exists.

Theorem: If c is a constant and $g_1(Y), g_2(Y), \dots, g_k(Y)$ are functions of a continuous r.v. Y, then:

(1)

(2)

(3)

- The proofs of the above theorems are similar to those in the discrete case.

Example (Variance): If Y is a continuous r.v., the variance of Y is

where $\mu = E(Y)$.

- The standard deviation of Y is

Note: If c is a constant, then

Example 1 again: Find $E(Y)$, $V(Y)$ and σ.

$E(Y) =$

$E(Y^2) =$

So

Example 2 again: Find $E(Y)$, $V(Y)$, and σ.

$E(Y) =$

$E(Y^2) =$

4.4 The Uniform Distribution

Example 1: Suppose a bus is supposed to arrive at a bus stop at 8:00 a.m. In reality, it is equally likely to arrive at any time between 8:00 and 8:10 a.m.

- Let Y be the number of minutes after 8:00 that the bus arrives.
- This is a simple example of a continuous r.v.

Defn: A r.v. Y has a uniform probability distribution on the interval $[\theta_1, \theta_2]$ {Shorthand: $Y \sim \text{Unif}(\theta_1, \theta_2)$} if its pdf is

provided $\theta_1 < \theta_2$.

- Note this is a ________ function of y on the support of Y.

Graph of uniform pdf:

- Simple geometry indicates the total area under the density is:

- Example 1: What is the probability the bus will arrive between 8:02 and 8:05?

- What is the probability it will arrive after 8:06?

- The cdf of a uniform r.v. is easy to derive.
- Clearly,

- This is a ______________ __________ function of y.

<u>Graph of cdf:</u>

Theorem (Uniform Mean and Variance):

If $Y \sim Unif(\theta_1, \theta_2)$, then:

Proof:

- Suppose the number of events Y occurring in a time interval $(0, t)$ follows a Poisson distribution. Then given that exactly one event occurs in $(0, t)$, the time T at which the event occurs follows a

Example 2: Suppose the number of customers per minute at McDonald's follows a Poisson distribution. If exactly one customer arrives between 7:00 and 7:05, then find the probability that this customer arrived between 7:02 and 7:04.

- The values θ_1 and θ_2 are called the parameters of the uniform distribution.
- The values of a density's parameters determine the specific form of the density.
- If $\theta_1 = 0$ and $\theta_2 = 1$, the Unif $(0,1)$ distribution is called the standard uniform distribution.