

4.11 Expected Values of Piecewise

Functions and Mixed Distributions

- We have seen examples of finding $E[g(Y)]$, the expected value of some function of a r.v. Y .
- Suppose $g(\cdot)$ is a piecewise function defined in a manner such as:

where A_1, A_2, \dots, A_k form a partition of the support of Y .

Note: The piecewise function $g(y)$ may be continuous, or it may not be.

- Then

- If Y is a discrete r.v., this formula is similar, with sums replacing the integrals (and the pmf replacing the pdf).

Example 1: Suppose a distributor ships a random amount Y (in thousands of gallons) of oil each day. This amount Y has pdf

$$f(y) = \begin{cases} -\frac{3}{32}(y^2 - 4y) & \text{if } 0 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$

The daily profit is 10 ¢ per gallon shipped, up to 3000 gallons. The profit rate increases by an additional 5 ¢ per gallon past 3000 gallons. Thus the profit function $g(Y)$ is:

Find the expected daily profit.

Example 2: An insurance policy reimburses fully any loss up to \$10,000. For losses exceeding \$10,000, the policy pays \$10,000 plus 80% of the excess loss. Suppose the loss Y (in thousands of dollars) has pdf

$$f(y) = \begin{cases} \frac{2}{y^3} & \text{if } y \geq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- For a random claim, what is the insurance company's expected payout?

Mixed Distributions

- A mixed distribution has probability mass placed at one or more discrete points, and has its remaining probability spread over intervals.

Example 3: Let Y be the annual amount paid out to a random customer by an auto insurance company. Suppose $\frac{3}{4}$ of customers each year do not receive any payout. Of those who do receive benefits, the payout amount $\frac{1}{2}$ follows a gamma(10, 160) distribution.

- For a random customer, what is the expected payout amount?

The expected payout is:

Note: If Y has a mixed distribution, the cdf of Y may be written as

where:

- In this case,

where Y_1 and Y_2 are the "appropriate" discrete and continuous r.v.'s, respectively.

- Also, for any function $g(Y)$ of Y :

Example 4: In a lab experiment, a rat is given a shot of a dangerous toxin. There is probability 0.2 that the rat will die instantly. If it survives the initial shot, suppose it would ordinarily then die at a random point in time over the next 24 hours. However, if it is still alive 6 hours after the shot, the rat is killed.

- Find the cdf and the expected value of the survival time for a rat entering this experiment.

- Denote the survival time by Y .

The discrete points are:

So define a discrete r.v. Y_1 such that

Then the discrete r.v. Y_1 has cdf

Graph:

And define a continuous r.v. Y_2 with cdf

i.e., a _____ cdf.

Note

So the cdf of Y is

And $E(Y) =$

-What is the probability that a rat entering the experiment will survive 3 hours or less?