The Gauss-Markov Theorem

- This famous theorem states that the least-squares estimators of β_0 and β_1 are BLUEs (Best Linear Unbiased Estimators), where "best" refers to having minimum variance.

Theorem: Consider the SLR model

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \quad i=1,\dots,n,$$

where the ε_i are independent with $E(\varepsilon_i)=0$ and $V(\varepsilon_i)=\sigma^2$. Then the least-squares estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ have the minimum variance among all linear unbiased estimators of β_0 and β_1.

Note: A linear estimator is one that is a linear function of $Y_1,\dots,Y_n$, i.e., can be written as $\sum_{i=1}^{n} k_i Y_i$ for constants $k_1,\dots,k_n$.

Proof (case of β_1):

- The proof that $\hat{\beta}_0$ is BLUE for β_0 is similar.