"Treatments" → factor levels (in one-way ANOVA)
or factor level combinations (in multi-factor ANOVA)

Design of Experiments

- Factorial experiments require a lot of resources

- Sometimes real-world practical considerations require us to design experiments in specialized ways.

- The design of an experiment is the specification of how treatments are assigned to experimental units.

Goal: Gain maximum amount of reliable information using minimum amount of resources.

- Reliability of information is measured by the standard error of an estimate.

- How to decrease standard errors and thereby increase reliability?

- Increase sample size (costly, sometimes impractical)

- Decrease population variance σ^2

How?

- Recall the One-Way ANOVA:

- Experiments we studied used the Completely Randomized Design (CRD).

Example 3: An industrial experiment is conducted over several days (with a different lab technician each day).

- **Possible block design:**

Then the technicians (or the days) could be blocks.

Example 4: (Table 10.2 data)
Y = wheat crop yield
experimental units = plots of wheat
treatments = 3 different varieties of wheat (A, B, C)
blocks = regions of field

Possible arrangement:

1	2	3	4	5
B	C	A	C	B
A	B	B	A	C
C	A	C	B	A

- **The data are given in Table 10.2.**

- **Note: Variety A has the greatest mean yield, but there is a sizable variation among blocks.**

- **If we had used a CRD, this variation would all be experimental error variance (inflates MSW).**

- **Analysis as CRD (ignoring blocks):**

$$F^* = \frac{49.217}{13.608} = 3.62 \qquad (\text{P-value} = .059)$$

So $\alpha = .05$, we do not conclude the mean yield significantly differs across the 3 varieties.

- **But … within each block, Variety A clearly has the greatest yield (RBD will account for this).**

Formal Linear Model for RBD

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \qquad i = 1, \dots, t \quad j = 1, \dots, b$$

treatment index   block index

- This assumes <u>one observation per treatment-block combination</u>.

Y_{ij} = response value for treatment i in block j
μ = an overall mean response
τ_i = effect of treatment i
β_j = effect of block j
ε_{ij} = random error term

- Looks similar to two-factor factorial model with one observation per cell. (assume no treatment × block interaction)

<u>Key difference</u>: With RBD, we are not equally interested in both factors.

- The treatment factor is of primary importance; the blocking factor is included merely to reduce experimental error variance.

- With RBD, the block effects are often considered random (not fixed) effects.

- This is true if the blocks used are a random sample from a large population of possible blocks.

• If treatment effects are fixed and block effects are random, the RBD model is called a <u>mixed model</u>.

• In this case, the treatment-block interaction is also random.

• This interaction measures the variation among treatment effects across the various blocks.

• The mean square for interaction is used here as an estimate of the <u>experimental error variance</u> σ^2.

Expected Mean Squares in RBD

Source	df	E(MS)
Trts	$t-1$	$\sigma^2 + \frac{b}{t-1}\sum_i \tau_i^2$
Blocks	$b-1$	$\sigma^2 + t\,\sigma_\beta^2$
Exper. Error (Trt x Block Interaction)	$(t-1)(b-1)$	σ^2

σ^2 = experimental error variance

σ_β^2 = variance among block effects

• **Testing for an effect on the mean response among treatments:**

H₀: $\tau_1 = \tau_2 = \cdots = \tau_t = 0 \iff \sum_i \tau_i^2 = 0$

• **The correct test statistic is apparent based on E(MS):**

$$F^* = \frac{MS(Trts)}{MSE}$$

Reject H₀ if: $F^* > F_\alpha[t-1, (t-1)(b-1)]$

• **Testing for significant variation across blocks:**

H₀: $\sigma_\beta^2 = 0$ $\qquad H_a: \sigma_\beta^2 > 0$

• **The correct test statistic is again apparent:**

$$F^* = \frac{MS(Blocks)}{MSE}$$

Reject H₀ if: $F^* > F_\alpha[(b-1), (t-1)(b-1)]$

Example: **(Wheat data – Table 10.2)**

• **The ANOVA table formulas are the same as for the two-way ANOVA.**

• **We use software for the ANOVA table computations.**

$H_0: \tau_1 = \tau_2 = \tau_3 = 0$

$H_a: \tau_1, \tau_2, \tau_3$ not all zero

RBD analysis (Wheat data):

F* = $\dfrac{MS(Trts)}{MSE} = \dfrac{49.217}{1.8} = 27.34$ (P-value $= .0003$)

- **We conclude that the mean yields are significantly different for the different varieties of wheat. At $\alpha = 0.05$, we reject $H_0: \tau_1 = \tau_2 = \tau_3 = 0$.**

<u>Note</u> (for testing about blocks): $H_0: \sigma_\beta^2 = 0$ vs. $H_a: \sigma_\beta^2 > 0$

F* = $\dfrac{MS(Blocks)}{MSE} = \dfrac{37.225}{1.8} = 20.68$ (P-value $= .0003$)

- **We would also reject $H_0: \sigma_\beta^2 = 0$ and conclude there is significant variation among block effects.**

- **We can again make pre-planned comparisons using contrasts.**

<u>Example</u>: Is Variety A <u>superior</u> to the other two varieties in terms of mean yield? $L = \mu_A - \frac{1}{2}\mu_B - \frac{1}{2}\mu_C$

H_0: $\mu_A - \frac{1}{2}\mu_B - \frac{1}{2}\mu_C = 0$

H_a: $\mu_A - \frac{1}{2}\mu_B - \frac{1}{2}\mu_C > 0$

Result: $t^* = 7.28$ (evidence in favor of H_a)

$t^* = 7.28 > 1.86 = t_{.05, 8 d.f.}$ $\Rightarrow$ reject H_0, conclude H_a.

SAS gives two-sided p-value of $< .0001$.

$\Rightarrow$ One-sided p-value here is $< \frac{.0001}{2} \Rightarrow < .00005$

$\rightarrow$ Reject H_0, conclude Variety A <u>is</u> superior in terms of mean yield.

• The estimate of σ^2 was MSW. This measured the variation among responses for units that were treated alike (measured variation within groups).

• We call this estimating the experimental error variation.

• What if we divide the units into subgroups (called blocks) such that units within each subgroup were similar in some way?

• We would expect the variation in response values among units treated alike within each block to be relatively small.

Randomized Block Design (RBD)

• RBD: A design in which experimental units are divided into subgroups called blocks and treatments are randomly assigned to units within each block.

• Blocks should be chosen so that units within a block are similar in some way.

• Reasons for the variation in our data values:

CRD (Chap. 6)

- Variation due to treatments (levels)
- Experimental error variation (leftover variation)

RBD (Chap. 10)

- Variation due to treatments
- Variation due to blocks
- Experimental error variation (leftover)

now reduced

variation

- **Benefits of a reduction in experimental error:**
  - **decreases MSW (denominator of F* ratios used in F-tests) → more power to reject null hypotheses**
  - **decreases standard errors of means → shorter CIs for mean responses**

Example 1: Suppose we investigate whether the average math-test scores of students from 8 different majors differ across majors.
- **But … students will be taught by different instructors.**
- **We're not as interested in the instructor effect, but we know it adds another layer of variability.**

Solution: Make "instructors" the blocks

units = students

(response) Y = test score

treatments = 8 majors

blocks = the instructors

Example 2: Lab animals of a certain species are given different diets to determine the effect of diet on weight gain.
- **Possible block design:**

units = animals

Y = weight gain

treatments = diets

blocks = litters the animals were born into