A 5×5 Latin Square is:

			Day		
		M	Tu	W	Th	F
Time	9-10	A	B	C	D	E
	10-11	B	C	D	E	A
	11-12	C	D	E	A	B
	1-2	D	E	A	B	C
	2-3	E	A	B	C	D

- **Each music type appears once on each day and once at each time of day.**

- **Testing for a significant effect of music type on mean productivity:**

$$\mathbf{F^*} = \frac{MS(Trt)}{MSE} = \frac{14.079}{1.147} = 12.27 \quad (\text{P-value} = .0003)$$

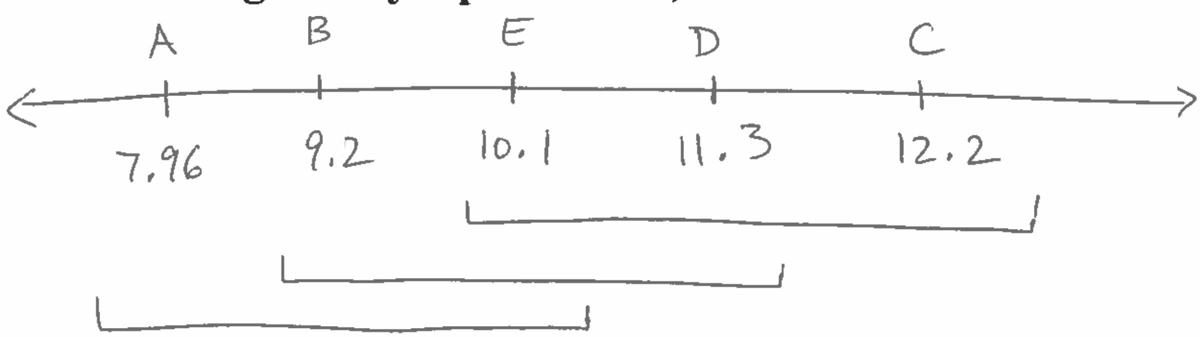
Note $F^* > F_{.05,4,12} = 3.26$ and P-value $< .05 \Rightarrow$ Reject H_0: all $\tau_k = 0$

- **There is a significant difference in mean productivity among the five music types.**

- **<u>Note</u>: There is also a significant row effect (time of day) and a significant column effect (day of week).**

- **Specifically, which music types are significantly different?**

- **Using Tukey's procedure, we see:**



Tukey results:

C significantly different from A and B

D significantly different from A.

<u>Summary</u>:

- **Main advantage of a Latin Square design:**

<u>Efficiency</u> – can perform useful tests with relatively few experimental units.

- **Main disadvantage: cannot test for any interaction.**

Other Linear Models

Recall: One-way ANOVA model equation:

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

SLR model equation:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

- **These seem quite different and are used in different data analysis situations.**

- **But these and other models can be unified. They are each examples of the general linear model.**

Dummy Variables

- **The one-way ANOVA model may be represented as a regression model by using dummy variables.**

Dummy variables (indicator variables): Take only the values 0 and 1 (sometimes -1 in certain contexts).

- **One-way ANOVA model (above) is equivalent to:**

$$Y = \mu X_0 + \tau_1 X_1 + \tau_2 X_2 + \cdots + \tau_t X_t + \varepsilon$$

where we define these dummy variables:

$X_0 = 1$ for all observations

$X_1 = \begin{cases} 1 & \text{for observations from group 1} \\ 0 & \text{otherwise} \end{cases}$

$X_2 = \begin{cases} 1 & \text{for observations from group 2} \\ 0 & \text{otherwise} \end{cases}$

.

.

this will change $\rightarrow$ $X_t = \begin{cases} 1 & \text{for observations from group } t \\ 0 & \text{otherwise} \end{cases}$

Example: Suppose we have a one-way analysis with two observations from level 1, two observations from level 2, and three observations from level 3. The X matrix of the "regression" would look like:

$$X = \begin{array}{c} \begin{array}{cccc} X_0 & X_1 & X_2 & X_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

[7 rows, 4 columns (for X_0, X_1, X_2, X_3)]

● **The *Y*-vector of response values and the vector of parameter estimates would be:**

$$\underset{\sim}{Y} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \\ Y_{31} \\ Y_{32} \\ Y_{33} \end{bmatrix}$$

vector of parameter estimates

$$\begin{bmatrix} \hat{\mu} \\ \hat{\tau}_1 \\ \hat{\tau}_2 \\ \hat{\tau}_3 \end{bmatrix} = (X^T X)^{-1} X^T \underset{\sim}{Y}$$

Problem: It turns out that X^TX is not invertible in this case.

- There are $t = 3$ non-redundant equations and $t + 1 = 4$ unknown parameters here.

- We fix this by adding one extra restriction to the parameters.

- Most common (we used this before): Force $\sum_{i=1}^{t} \tau_i = 0$

by defining $\tau_t = -\tau_1 - \ldots - \tau_{t-1}$.

- Using this approach, we need $t - 1$ dummy variables to represent t levels.

- If an observation comes from the <u>last</u> level, it gets a value of -1 for <u>all</u> dummy variables $X_1, X_2, \ldots, X_{t-1}$.

X matrix from previous data set using this approach:

7 rows, 3 columns

$$X = \begin{array}{c} \begin{array}{ccc} X_0 & X_1 & X_2 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \end{array}$$

parameter estimates:

$$\begin{bmatrix} \hat{\mu} \\ \hat{\tau}_1 \\ \hat{\tau}_2 \end{bmatrix}$$

Now $\hat{\tau}_3 = -\hat{\tau}_1 - \hat{\tau}_2$

Again, this vector of parameter estimates is obtained by $(X^TX)^{-1}(X^TY)$