STAT 520 – Homework 1B – Fall 2025

3) Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance $\sigma_e^2$. Let $\{Y_t\}$ be a process defined as:
$Y_t = e_t + 0.2e_{t-1}$.
a) Find the autocovariance function *and* autocorrelation function of $Y_t$.
[Hint: Calculate $\text{cov}(Y_t, Y_{t-k})$ case-by-case for several values of $k$ (i.e., for $k = 0$, $k = 1$, $k \geq 2$).] Show all your steps clearly.
b) Is the time series $\{Y_t\}$ stationary? Explain your answer.

4) Apply a moving average filter to $Y_t$, where $Y_t$ is the natural logarithm of the Johnson and Johnson earnings data (the original data are given in the `jj` object in the `astsa` package). Specifically, let
$V_t = (Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}) / 4$. The R code

```
v = filter(y, rep(1/4, 4), sides = 1)
```

may be helpful in implementing this. Type `help(filter)` in R for more details about this R function. Plot $Y_t$ as a line and overlay (superimpose) $V_t$ as a dashed line, and provide this plot. Discuss whether the moving average filter captures the overall trend in the time series.

5) [Required for graduate students, extra credit for undergraduate students] Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance $\sigma_e^2$. Let $\{Y_t\}$ be a process defined as: $Y_t = e_t e_{t-1}$. Showing all your steps, find the mean function and the autocovariance function of $Y_t$. [Hint: Use facts about the expected value of the product of independent random variables.] Is the time series $\{Y_t\}$ stationary? Explain your answer.