5.

$$\frac{\mathrm{d}}{\mathrm{d}\mu}S_c(\phi,\mu) = \sum_{t=2}^{n}\frac{\mathrm{d}}{\mathrm{d}\mu}[(Y_t-\mu)-\phi Y_{t-1}+\phi\mu]^2$$

$$\Rightarrow \sum_{t=2}^{n} 2[Y_t-\mu-\phi Y_{t-1}+\phi\mu][-1+\phi]=0$$

$$\Rightarrow \sum_{t=2}^{n} -2[(1-\phi)(Y_t-\phi Y_{t-1}-(1-\phi)\mu)]=0$$

$$\Rightarrow -2(1-\phi)\sum_{t=2}^{n}[Y_t-\phi Y_{t-1}-(1-\phi)\mu]=0$$

$$\Rightarrow \sum_{t=2}^{n} Y_t-\phi\sum_{t=2}^{n} Y_{t-1}-(n-1)(1-\phi)\mu=0$$

$$\Rightarrow \sum_{t=2}^{n} Y_t-\phi\sum_{t=2}^{n} Y_{t-1}=(n-1)(1-\phi)\mu$$

$$\Rightarrow \hat{\mu}_{LS}=\frac{1}{(n-1)(1-\phi)}\left[\sum_{t=2}^{n} Y_t-\phi\sum_{t=2}^{n} Y_{t-1}\right]$$

6.

$$\frac{\mathrm{d}}{\mathrm{d}\phi}S_c(\phi,\mu) = \sum_{t=2}^{n}\frac{\mathrm{d}}{\mathrm{d}\phi}[(Y_t-\bar{Y})-\phi(Y_{t-1}-\bar{Y})]^2$$

$$\Rightarrow \sum_{t=2}^{n} 2[(Y_t-\bar{Y})-\phi(Y_{t-1}-\bar{Y})][-(Y_{t-1}-\bar{Y})]=0$$

$$\Rightarrow -2\sum_{t=2}^{n}[(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})-\phi(Y_{t-1}-\bar{Y})^2]=0$$

$$\Rightarrow \sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})-\phi\sum_{t=2}^{n}(Y_{t-1}-\bar{Y})^2=0$$

$$\Rightarrow \sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})=\phi\sum_{t=2}^{n}(Y_{t-1}-\bar{Y})^2$$

$$\Rightarrow \hat{\phi}_{LS}=\frac{\sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=2}^{n}(Y_{t-1}-\bar{Y})^2}$$

7. Taking the natural logarithm of the likelihood function and writing $v=\sigma_e^2$, we have

$$\log L(\phi,\mu,v) = -\frac{n}{2}\log(2\pi v)+\log(1-\phi^2)^{1/2}-\frac{1}{2v}S(\phi,\mu)$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}v}\log L(\phi,\mu,v) = -\frac{n}{2}\left(\frac{2\pi}{2\pi v}\right)+\frac{1}{2v^2}S(\phi,\mu)=0$$

$$\Rightarrow -\frac{n}{2v} + \frac{S(\phi, \mu)}{2v^2} = 0$$

$$\Rightarrow -nv + S(\phi, \mu) = 0$$

$$\Rightarrow S(\phi, \mu) = nv$$

$$\Rightarrow \hat{v}_{ML} = \hat{\sigma}_e^2 = \frac{S(\hat{\phi}, \hat{\mu})}{n}$$