STAT 520 – Homework 4A – Fall 2025

1) A data set of 57 consecutive measurements from a machine tool are in the `deere3` object in the `TSA` package. Type `library(TSA); data(deere3); print(deere3)` in R to see the data set.

(a) Estimate the parameters of a (mean-centered) AR(1) model for this series. Use the least squares method and maximum likelihood, and report the estimated parameters from each of these methods. Comment on any similarities and differences.

Using least squares, the estimate of ϕ is 0.533 and the estimate of μ is 160.08. The estimate of the noise variance is 2100808. Using maximum likelihood, the estimate of ϕ is 0.526 and the estimate of μ is 124.35. The estimate of the noise variance is 2069354. We see that using the two methods, the estimates of ϕ and the noise variance are fairly similar, but there is a big difference in the estimate of the overall mean μ.

(b) Estimate the parameters of a (mean-centered) AR(2) model for this series. Use the least squares method and maximum likelihood, and report the estimated parameters from each of these methods. Comment on any similarities and differences.

Using least squares, the estimate of ϕ_1 is 0.525, the estimate of ϕ_2 is 0.008, and the estimate of the mean μ is 201.2. The estimate of the noise variance is 2118086. Using maximum likelihood, the estimate of ϕ_1 is 0.521, the estimate of ϕ_2 is 0.008, and the estimate of the mean μ is 123.24. The estimate of the noise variance is 2069209. We see that using the two methods, the estimates of ϕ_1 and ϕ_2 and the noise variance are fairly similar, but there is a big difference in the estimate of the μ term.

(c) Compare the results of the ML fits from parts (a) and (b). Which model do you believe is preferable? Briefly explain your answer.

Comparing the ML fits, the AIC is better for the AR(1) model, at 995.02, compared to the AR(2) model at 997.02.

3) A data set of 324 measurements of an industrial robot's positions are in the `robot` object in the `TSA` package. Type `library(TSA); data(robot); print(robot)` in R to see the data set.

(a) Estimate the parameters of a (mean-centered) AR(1) model for these data, using maximum likelihood. Give the equation of the estimated model.

Using maximum likelihood, the estimate of ϕ is 0.308 and the estimate of μ is 0.0015. The estimate of the noise variance is 0.00000648. The equation of the estimated model is
$(Y_t - 0.0015) = 0.308\,(Y_{t-1} - 0.0015) + e_t$, with estimated noise variance 0.00000648.

(b) Give an approximate 95% confidence interval for ϕ, the coefficient in the AR(1) model.
$0.308 \pm 1.96[(1 - 0.308^2)/324]^{1/2} = 0.308 \pm 1.96(0.0528) = (0.204, 0.411)$

(c) Estimate the parameters of an IMA(1,1) model for these data. Give the equation of the estimated model.
Using maximum likelihood, the estimate of θ is –0.8713 in R's formulation, or 0.8713 in our book's formulation. The equation of the estimated model is
$Y_t = Y_{t-1} + e_t - 0.8713e_{t-1}$, with estimated noise variance 0.00000607, or
$\nabla Y_t = e_t - 0.8713e_{t-1}$, with estimated noise variance 0.00000607, where $\nabla Y_t = Y_t - Y_{t-1}$

(d) Note that the output labeled "aic" given by the arima() function output uses a slightly different definition of AIC than the AIC() function in R does. Verify this by reporting both the "aic" output and the AIC()result for the models in parts (a) and (c). However, as long as you use the same definition when comparing models, you can use either definition (since the formulas only differ by a constant). But why is it not recommended to compare the models in parts (a) and (c) using AIC?

The AR(1) model has an "aic" of -2947.08. The IMA(1,1) model has an "aic" of -2959.9. The AIC() function gives -2945.078 for the AR(1) model and -2957.901 for the IMA(1,1) model. However, the AR(1) model and the IMA(1,1) model should not be compared using AIC since they involve different levels of differencing, and thus the two models' response variables are inherently different.