STAT 521 - Applied Stochastic Processes

- A stochastic process is a collection of random variables indexed by time.
- We begin with a review of probability.

Chapter 1: Review of Probability Theory

Defn: An experiment is a random phenomenon whose outcome is not predictable in advance.

Defn: The sample space (denoted S) of an experiment is the set of all possible outcomes.

Example 1: Consider the experiment of rolling 2 dice. The sample space consists of pairs (i, j) such that

Example 2: Consider the experiment of measuring the lifetime of a part. The sample space is:

Defn. Any subset E of the sample space S is called an event.

Example 1: Let E be the event that the sum of the 2 dice is prime.

Then $E =$

Example 2: Let E be the event that the part lasts between 2 and 4 years. Then $E =$

Defn. The union of two events E and F is denoted and is the set of outcomes that belong to E or to F (or to both).

Defn. The intersection of two events E and F is denoted

and is the set of outcomes that belong to both E and F.

Example 1: Let F be the event that the sum of the 2 dice is even.

Then $EF =$

Note: If no outcomes belong to both E and F, we say that $EF = \emptyset$ (the null event) and that E and F are mutually exclusive.

- We can consider unions and intersections of a sequence of events $\{E_n\}$, $n = 1, 2, \ldots$:

$\bigcup_{n=1}^{\infty} E_n$ is

$\bigcap_{n=1}^{\infty} E_n$ is

<u>Defn.</u> The <u>complement</u> of E, denoted E^c, is the set of outcomes in S that do not belong to E.

<u>Note</u>: $S^c =$

1.3 Probabilities of Events

- The probability of event E, denoted $P(E)$, satisfies the conditions:

(i)

(ii)

(iii) For any sequence of events $E_1, E_2, \ldots$ such that $E_n E_m = \emptyset$ for $n \neq m$, then:

- When outcomes are <u>equally</u> <u>likely</u>, probabilities can often be readily calculated in simple problems:

<u>Example 1</u>: $P(E) =$

Basic Probability Rules

Complement Rule: $P(E^c) =$

Additive Rule:

Extended Additive Rule:

Inclusion-Exclusion Identity:

1.4 Conditional Probabilities

- The conditional probability of E given F is denoted $P(E|F)$ and is the probability that our outcome is in E, given the fact that our outcome is known to be in F.

Conditional Probability Formula:

Example 1: It can be shown that $P(F) = \frac{1}{2}$ (verify). Then the probability that the sum is prime, given that it is even, is:

-The probability the sum is even, given that it is prime, is:

Multiplicative Rule:

1.5 Independent Events

Defn: Two Events E and F are independent if and only if

Note: (*) is equivalent to:

and also equivalent to:

- So E and F are independent if and only if (*), (**), or (***) are true (they will all be true or will all be false).

Example 1: Are E and F independent?

Independence of More than Two Events

- Events $E_1, E_2, \ldots, E_n$ are (jointly) independent if for any subset $E_{1'}, E_{2'}, \ldots, E_{r'}$, $r \le n$:

<u>Example 3</u>: Consider an urn with four balls labeled 1, 2, 3, and 4. We draw a ball at random. Define the events:

$E = \{1, 2\} \quad F = \{1, 3\} \quad G = \{1, 4\}$

Then

So E, F, G are

- But

So E, F, and G are

1.6 Bayes' Formula

- If E and F are two events, note

More Than Two Events:

- Suppose $F_1, F_2, \ldots, F_N$ are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$. Then

Bayes' Formula:

Example (#39): Stores A, B, C have 50, 75, and 100 employees and, respectively, 50%, 60%, and 70% of these are women. One employee resigns and she is a woman. What is the probability she works in store C? Assume resignations are equally likely among all employees, regardless of sex.

Let $E =$

$F_1 =$

$F_2 =$

$F_3 =$

$P(\qquad) =$

Note