- The joint pmf of two discrete r.v.'s X and Y is

- The joint pdf of two continuous r.v.'s X and Y is $f(x,y)$ such that

for any subsets A and B of the real line.

- The marginal pmf (or marginal pdf) of X or Y may be found by summing (or integrating) the joint pmf (or joint pdf) over all possible values of the <u>other</u> r.v.:

$E[g(X,Y)] =$

Example: For any constants a, b:

- Joint distributions may be defined analogously for more than two r.v.'s and the results follow similarly. For example, for r.v.'s $X_1, \ldots, X_n$ and constants $a_1, \ldots, a_n$:

Independent Random Variables

Defn. Two r.v.'s X and Y are independent if for all a, b:

- This condition is equivalent to:

which is equivalent to

- If X and Y are independent, then for any functions g and h:

- In particular, if X and Y are independent.

<u>Defn</u>. The <u>covariance</u> between two r.v.'s X and Y is

<u>Note</u>: If X and Y are independent, then $\operatorname{cov}(X, Y) =$ (The converse is not always true.)

Properties of Variances and Covariances

① $\text{cov}(X, X) =$

② $\text{cov}(X, Y) =$

③ $\text{cov}(aX, Y) =$

④ $\text{cov}(X, Y+Z) =$

⑤ $\text{cov}\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) =$

⑥ $\text{var}\left(\sum_{i=1}^{n} X_i\right) =$

- If $X_1, \dots, X_n$ are independent r.v.'s, then

Independence and mgf's

- The mgf of a <u>sum</u> of independent r.v.'s is the <u>product</u> of the individual mgf's.
- This can be used to show the following facts:

- The above facts extend similarly to sums of more than two independent r.v.'s.

Important Limit Theorems

Markov's Inequality: If X is a nonnegative r.v., then for any $a > 0$:

Chebyshev's Inequality: If X is a r.v. with mean μ and variance σ^2, then for any $k > 0$:

<u>Law of Large Numbers</u>: If $X_1, X_2, \ldots$ are a sequence of independent and identically distributed (iid) r.v.'s with $E(X_i) = \mu$, then with probability 1,

<u>Central Limit Theorem</u>: If $X_1, X_2, \ldots$ are a sequence of iid r.v.'s with mean μ and variance σ^2 for each X_i, then

converges in distribution to standard normal $(N(0,1))$ as $n \to \infty$.

2.9 Stochastic Processes

<u>Defn</u>: A <u>stochastic process</u>, denoted $\{X(t), t \in T\}$, is a collection of r.v.'s indexed by t (which often represents time).

- At each value t, the r.v. $X(t)$ is the <u>state</u> of the process at time t.

- The set T is the <u>index set</u> of the process.
- If T is a <u>countable</u> set, then $\{X(t)\}$ is a <u>discrete-time process</u>.
- If T is an interval of the real line, then $\{X(t)\}$ is a <u>continuous-time process</u>.
- The <u>state space</u> of the process is the set of all possible values that $\{X(t)\}$ can take.

<u>Examples:</u> 1 $X(t)$ = number of customers in a store at time t

2 $X(t) =$