Chapter 3: Conditional Distributions

<u>Defn</u>. For discrete r.v.'s X and Y, the <u>conditional</u> <u>pmf</u> of X given $Y=y$ is:

- The conditional expectation of X given $Y=y$ is

<u>Defn</u>. For continuous r.v.'s X and Y, the <u>conditional</u> <u>pdf</u> of X given $Y=y$ is:

- The conditional expectation of X given $Y=y$ is:

3.4   Iterated Expectations

Theorem: If X and Y are r.v.'s, then $E(X) = E[E(X|Y)]$

Note: The inner expectation is with respect to the conditional distribution of $X|Y$ and the outer expectation is with respect to the distribution of Y.

Conditional Variance

$$var(X) = E[var(X|Y)] + var[E(X|Y)]$$

Example 1: Suppose the number of accidents per year for each insurance policyholder is Poisson with some mean that is itself random and follows a gamma $(1,5)$ distribution across policyholders. For a randomly chosen policyholder, what is the expected number of accidents?

- What is the variance of the number of accidents?

3.5   Computing Probabilities by Conditioning

- The connection between the probability of an event and the expected value of an indicator variable allows us to compute probabilities by conditioning.

- Let E denote some event of interest.

- Let the indicator X be such that

$$X =$$

- Then $E(X) =$

and $E[X|Y=y] =$

for any r.v. Y.

- So by the law of iterated expectations,

$$P(E) = \begin{cases} \\ \\ \\ \\ \end{cases}$$

Example #3.21: If X and Y are independent continuous r.v.'s, then we may find $P(X < Y)$ by defining $E =$ "$X < Y$".

Example 1 again: What is the probability that the random policyholder has n accidents in the next year (where $n = 0, 1, 2, \ldots$) ?

Example (#66): The opponents of a soccer team are of two types: class $\underline{1}$ or class 2. The number of goals our team scores against a class i opponent is Poisson with mean λ_i, where $\lambda_1 = 2$, $\lambda_2 = 3$. This weekend our team will play two games. Assume the first opponent is a class 1 team with probability 0.6, and the second is (independently) a class $\underline{1}$ team with probability 0.3.

- What is the expected number of goals our team will score this weekend?

- what is the probability our team will score a total of 5 goals this weekend?