In R, we can compute:

Recall Example 3: If at some point, the first urn has 2 white balls, what is the probability that, after four further steps, the first urn will have less than 2 white balls?

Defining A Special Set of States

- Suppose A is a specified set of states, and we are interested in the Markov chain entering any state in A by time m.

- The probability of this is

- We define a new Markov chain $\{W_n\}$ whose states consist of:

- Let $\{P_{ij}\}$ be the transition probabilities for the original chain $\{X_n\}$. Define

i.e., N is the

- Then the new chain is defined by

- So $\{W_n\}$ is in the same state as $\{X_n\}$ up until $\{X_n\}$ enters A, at which point $\{W_n\}$ goes to state A forever.
- Then $\{W_n\}$ is a Markov chain with transition probabilities $\{Q_{ij}\}$, where

Now,

Example 4: Consider repeated independent rolls of two fair dice. We want the probability that the first run of 3 straight "prime sum" results occurs within the first 10 rolls.

Define a Markov chain with states

- The transition probability matrix of this chain is:

$Q =$

- Since we begin in state , the probability of getting to state A within 10 steps is:

- What is the probability that the first run of 3 straight prime-sums is completed on the 10th roll?

Unconditional Probabilities

- Note P_{ij}^{n} is a conditional probability — the probability of being in state j at time n given that we begin in state i.
- What about the unconditional probability of being in state j at time n (not knowing the initial state)?

- First we consider the probability distribution of the initial state:

- Then the unconditional probability

Example 1: If $\alpha_0 = 0.3$, $\alpha_1 = 0.7$, then the probability of rain five days from some unspecified day is:

Example 3: Suppose someone arranges the balls at random into the two urns (two balls in each urn) before we start. What is the probability that after 4 steps, the first urn will have 2 white balls?

Note

Similarly, we can show that

thereby deriving the entire unconditional distribution of X_4.