- Find the expected number of families checked in at the beginning of day n if there are initially i families checked in.

- What are the stationary probabilities π_j of $\{X_n\}$?

Limiting Probabilities

Recall Example 1: A two-state Markov chain with transition probability matrix

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}$$

Note

$P^4 =$

$P^8 =$

$P^{12} =$

- It appears P_{ij}^n converges to some fixed value as $n \to \infty$, not depending on the initial state i.

- Such limiting probabilities do not always exist.

Example: Consider a two-state Markov chain with transition probability matrix

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- This chain continually

But $P_{00}^n =$

- So P_{00}^n

Defn. An irreducible chain is periodic if it can only return to a state in a multiple of $d > 1$ steps.

- In the previous example, the period $d =$.

- An irreducible chain that is not periodic is called <u>aperiodic</u>.

<u>Theorem:</u> For any aperiodic Markov chain, the limiting probabilities:

(i) exist;

(ii) do not depend on the initial state;

(iii) equal the long-run proportions, i.e.,

<u>Proof</u>: (part (iii)):

Application: (The Gambler's Ruin Problem)

- This is related to the Random Walk process discussed earlier.
- Suppose a gambler makes successive and independent bets, each time having probability p of winning \$1 and probability $q = 1-p$ of losing \$1. Let X_n = the gambler's fortune at time n. The gambler's goal is to amass N dollars; what is the probability he will reach his goal before going broke?
- The classes are:

and the transition probabilities are

- The absorbing classes are ______________ and the other class is ______________.
- Each ______________ state is visited only finitely often, so eventually:

- Let P_i be the probability that, if the gambler begins with i dollars, he eventually reaches his goal of \$N.
- In Section 4.5.1, it is shown that:

Note:

Example: A roulette player has chance $\frac{18}{38}$ of winning each bet. He enters the casino with \$60 and will play successive \$10 bets until his fortune reaches \$100 or he goes broke. What is the probability that he goes broke?