4.6 Mean Time Spent in Transient States

- Consider a finite state Markov chain.
- Number the states so that

is the set of transient states.
- Let

be the matrix with transition probabilities from transient states into transient states.
- This is not a full transition probability matrix (its rows do not all sum to 1).
- For transient states i and j, let S_{ij} be the expected number of time periods that the chain is in j, given that it starts in i.

- Define $\delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$

Then

- Note we need to sum over only the transient states $k=1,\ldots,t$ since the chain cannot go from a recurrent state to a transient state.

Why not?

Let the matrix S contain all $\{s_{ij}\}$, $i,j=1,\ldots,t$:

$S =$

- Note that in matrix notation,

- So given P_T, the $\{S_{ij}\}$ values are easily calculated.

<u>Roulette Example again</u>: What is the expected amount of time the gambler has \$80?

- Note letting 1 unit = \$10,

$P_T =$

- It is easy to calculate $(I-P_T)^{-1}$ in R.
- We want

- What is the expected amount of time the gambler has \$20?

- Note for transient states i and j, f_{ij} is the probability the chain ever enters j given that it starts in i.
- Conditioning on whether the chain enters j, we have:

- These probabilities can be found from S, which can be found from P_T.

<u>Previous Example:</u> What is the probability the roulette gambler ever has exactly \$90?

- What is the expected amount of time the gambler plays the game?