Types of Events

- Suppose $\{N(t)\}$ is a Poisson Process and each time an event occurs, it can be classified into two types (Type I or Type II). The events' classifications are independent, and $P[\text{Type I}] = p$ for each event.
- Let $N_1(t)$ and $N_2(t)$ represent the numbers of Type I and Type II events occurring in $[0, t]$.
- Clearly $N_1(t) + N_2(t) =$

Theorem: $\{N_1(t)\}$ and $\{N_2(t)\}$ are independent Poisson processes with respective rates λp and $\lambda(1-p)$.

Proof: (1)

(2)

(3)

(4)

- So $\{N_1(t)\}$ is a Poisson process, and, similarly, so is $\{N_2(t)\}$. Also, consider the joint pmf:

Example 1: If 70% of the store's customers are female, what is the probability that 3 or fewer male customers arrive in the first 15 minutes?

- This result generalizes: If events are of r types, each with probability $p_1, \dots, p_r$, then the $\{N_i(t)\}$ are independent Poisson processes with rates $\lambda p_1, \dots, \lambda p_r$.

Conditional Distribution of Arrival Times

- In a Poisson process, given that exactly one event has occurred by time t, what is the distribution of that event's time?
- We know the event's time $T_1 \in$

For $0 \leq s \leq t$,

- This is the cdf of a ____________ r.v., so the event time $T_1 \sim$

<u>Fact</u>: If $Y_1, \dots, Y_n \overset{iid}{\sim} \text{Unif}(0,t)$, then the joint pdf of the order statistics $Y_{(1)} < Y_{(2)} < \cdots < Y_{(n)}$ is

- Let us consider, given $N(t) = n$, the conditional distribution of the n arrival times $S_1, \dots, S_n$:

- So we have shown that given that n events have occurred by time t, the arrival times $S_1, \dots, S_n$ have the same distribution as the ________ ________ from a ________ random sample.
- In other words, the <u>unordered</u> arrival times are ________ and ________ distributed on

<u>Example #5.21</u>: (Discounted cost of insurance claims) Suppose the times of insurance claims follow a

Poisson process with rate λ. The claim amounts are independent r.v.'s following a distribution with mean μ, and these amounts are independent of the claim times.

Let $S_i =$

$C_i =$

- Then the total discounted cost of all claims up to time t is

where α is the discount rate and $N(t)$ is the number of claims by time t. Find the expected total discounted cost.

Given $N(t) = n$, then $S_1, \ldots, S_n$ have the same distribution as