STAT 704 Test 1 Formula Sheet

Basics:

$$\mathrm{var}(Y) = E\{[Y - E(Y)]^2\} = E(Y^2) - [E(Y)]^2$$

$$\mathrm{cov}(Y,Z) = E\{[Y - E(Y)][Z - E(Z)]\} = E(YZ) - E(Y)E(Z)$$

$$\mathrm{corr}(Y,Z) = \frac{\mathrm{cov}(Y,Z)}{\sqrt{\mathrm{var}(Y)\,\mathrm{var}(Z)}}$$

One-Sample Formulas:

$$s^2 = \frac{\sum (Y_i - \bar{Y})^2}{n-1} \qquad \bar{Y} \pm t_{(1-\alpha/2,\, n-1)} \frac{s}{\sqrt{n}}$$

$$t^* = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}}$$

Two-Sample Formulas:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Equal-variances case:

$$(\bar{Y}_1 - \bar{Y}_2) \pm t_{(1-\alpha/2,\, n_1+n_2-2)} \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

$$t^* = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Unequal-variances case: Just use s_1^2, s_2^2 instead of s_p^2, s_p^2, and use approximation formula for degrees of freedom.

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Simple Linear Regression Formulas

$$b_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = \frac{\sum X_i Y_i - \frac{1}{n}\sum X_i \sum Y_i}{\sum X_i^2 - \frac{1}{n}\left(\sum X_i\right)^2}$$

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$e_i = Y_i - \hat{Y}_i \qquad S^2 = MSE = \frac{SSE}{n-2} = \frac{\sum(Y_i - \hat{Y}_i)^2}{n-2}$$

$$b_1 \pm t_{(1-\alpha/2,\, n-2)}\sqrt{\frac{MSE}{\sum(X_i - \bar{X})^2}} \qquad t^* = \frac{b_1}{\sqrt{\frac{MSE}{\sum(X_i - \bar{X})^2}}}$$

$$\hat{Y}_h \pm t_{(1-\alpha/2,\, n-2)}\sqrt{MSE\left[\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]}$$

$$\hat{Y}_h \pm t_{(1-\alpha/2,\, n-2)}\sqrt{MSE\left[1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]}$$

$$SSTO = \sum(Y_i - \bar{Y})^2 \qquad SSR = \sum(\hat{Y}_i - \bar{Y})^2$$

$$SSE = \sum(Y_i - \hat{Y}_i)^2 \qquad F^* = \frac{MSR}{MSE}$$

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} \qquad r = [\text{sign}(b_1)]\sqrt{R^2}$$

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Multiple Regression Formulas:

$$\underset{\sim}{b} = (X'X)^{-1}X'\underset{\sim}{Y} \qquad \hat{\underset{\sim}{Y}} = X\underset{\sim}{b} = H\underset{\sim}{Y}$$

$$H = X(X'X)^{-1}X' \qquad \underset{\sim}{e} = \underset{\sim}{Y} - \hat{\underset{\sim}{Y}} = \underset{\sim}{Y} - X\underset{\sim}{b} = (I - H)\underset{\sim}{Y}$$

$$MSR = \frac{SSR}{p-1}, \quad MSE = \frac{SSE}{n-p} \quad (\text{where } p = k+1)$$

$$F^* = \frac{MSR}{MSE}, \quad R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO}$$

$$R_a^2 = 1 - \frac{SSE/(n-p)}{SSTO/(n-1)}$$

For a constant matrix A, $E(A\underset{\sim}{Y}) = A\,E(\underset{\sim}{Y})$

$$\text{var}(A\underset{\sim}{Y}) = A\,\text{var}(\underset{\sim}{Y})\,A'$$

CI for β_j:

$$b_j \pm t_{(1-\alpha/2,\, n-p)}\sqrt{MSE\, c_{jj}}$$

t-test for $H_0: \beta_j = 0$

$$t^* = \frac{b_j}{\sqrt{MSE\, c_{jj}}},$$ where c_{jj} = j-th diagonal element of $(X'X)^{-1}$

CI for $E(Y_h)$ at a set of X-values $\underset{\sim}{X}_h$:

$$\hat{Y}_h \pm t_{(1-\alpha/2;\, n-p)}\sqrt{MSE\, \underset{\sim}{X}_h'(X'X)^{-1}\underset{\sim}{X}_h}$$

PI for $Y_{h(new)}$ at $\underset{\sim}{X}_h$.

$$\hat{Y}_h \pm t_{(1-\alpha/2;\, n-p)}\sqrt{MSE\left[1 + \underset{\sim}{X}_h'(X'X)^{-1}\underset{\sim}{X}_h\right]}$$