Test 1 Formula Sheet, Page 2

Simple Linear Regression Formulas

$$b_1 = \frac{S_{xy}}{S_{xx}} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{\sum X_i Y_i - \frac{1}{n}\sum X_i \sum Y_i}{\sum X_i^2 - \frac{1}{n}\left(\sum X_i\right)^2}$$

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$e_i = Y_i - \hat{Y}_i \qquad s^2 = MSE = \frac{SSE}{n-2} = \frac{\sum (Y_i - \hat{Y}_i)^2}{n-2}$$

$$b_1 \pm t_{(1-\alpha/2,\, n-2)} \sqrt{\frac{MSE}{\sum (X_i - \bar{X})^2}} \qquad t^* = \frac{b_1}{\sqrt{\frac{MSE}{\sum (X_i - \bar{X})^2}}}$$

$$\hat{Y}_h \pm t_{(1-\alpha/2,\, n-2)} \sqrt{MSE\left[\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right]}$$

$$\hat{Y}_h \pm t_{(1-\alpha/2,\, n-2)} \sqrt{MSE\left[1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum (X_i - \bar{X})^2}\right]}$$

$$SSTO = \sum (Y_i - \bar{Y})^2 \qquad SSR = \sum (\hat{Y}_i - \bar{Y})^2$$

$$SSE = \sum (Y_i - \hat{Y}_i)^2 \qquad F^* = \frac{MSR}{MSE}$$

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} \qquad r = [\text{sign}(b_1)]\sqrt{R^2}$$

STAT 704 · Formula Sheet

Multiple Regression Formulas:

$$\underset{\sim}{b} = (X'X)^{-1}X'\underset{\sim}{Y} \qquad \hat{\underset{\sim}{Y}} = X\underset{\sim}{b} = H\underset{\sim}{Y}$$

$$H = X(X'X)^{-1}X' \qquad \underset{\sim}{e} = \underset{\sim}{Y} - \hat{\underset{\sim}{Y}} = \underset{\sim}{Y} - X\underset{\sim}{b} = (I-H)\underset{\sim}{Y}$$

$$MSR = \frac{SSR}{p-1}, \quad MSE = \frac{SSE}{n-p} \qquad (\text{where } p = k+1)$$

$$F^* = \frac{MSR}{MSE}, \qquad R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO}$$

$$R_a^2 = 1 - \frac{SSE/(n-p)}{SSTO/(n-1)}$$

For a constant matrix A, $E(A\underset{\sim}{Y}) = A\,E(\underset{\sim}{Y})$

$$\text{var}(A\underset{\sim}{Y}) = A\,\text{var}(\underset{\sim}{Y})\,A'$$

CI for β_j:

$$b_j \pm t_{(1-\alpha/2,\, n-p)}\sqrt{MSE\, c_{jj}}$$

t-test for $H_0: \beta_j = 0$

$$t^* = \frac{b_j}{\sqrt{MSE\, c_{jj}}}, \text{ where } c_{jj} = j\text{-th diagonal element of } (X'X)^{-1}$$

CI for $E(Y_h)$ at a set of X-values $\underset{\sim}{X}_h$:

$$\hat{Y}_h \pm t_{(1-\alpha/2;\, n-p)}\sqrt{MSE\, \underset{\sim}{X}_h'(X'X)^{-1}\underset{\sim}{X}_h}$$

PI for $Y_{h(new)}$ at $\underset{\sim}{X}_h$:

$$\hat{Y}_h \pm t_{(1-\alpha/2;\, n-p)}\sqrt{MSE\left[1 + \underset{\sim}{X}_h'(X'X)^{-1}\underset{\sim}{X}_h\right]}$$