

STAT 704 Test 2 Formula Sheet

Multiple Regression Formulas:

$$\underline{\hat{b}} = (X'X)^{-1} X' \underline{y} \quad \underline{\hat{y}} = X \underline{\hat{b}} = H \underline{y}$$

$$H = X(X'X)^{-1} X' \quad \underline{e} = \underline{y} - \underline{\hat{y}} = \underline{y} - X \underline{\hat{b}} = (I - H) \underline{y}$$

$$MSR = \frac{SSR}{p-1}, \quad MSE = \frac{SSE}{n-p} \quad (\text{where } p = k+1)$$

$$F^* = \frac{MSR}{MSE}, \quad R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO}$$

$$R_a^2 = 1 - \frac{SSE/(n-p)}{SSTO/(n-1)}$$

For a constant matrix A , $E(A \underline{y}) = A E(\underline{y})$
 $\text{var}(A \underline{y}) = A \text{var}(\underline{y}) A'$

CI for β_j : $b_j \pm t_{(1-\alpha/2, n-p)} \sqrt{MSE c_{jj}}$

t-test for $H_0: \beta_j = 0$ $t^* = \frac{b_j}{\sqrt{MSE c_{jj}}}$, where $c_{jj} = j$ -th diagonal element of $(X'X)^{-1}$

CI for $E(Y_h)$ at a set of X-values \underline{X}_h :

$$\hat{Y}_h \pm t_{(1-\alpha/2, n-p)} \sqrt{MSE \underline{X}_h' (X'X)^{-1} \underline{X}_h}$$

PI for $Y_{h(\text{new})}$ at \underline{X}_h .

$$\hat{Y}_h \pm t_{(1-\alpha/2, n-p)} \sqrt{MSE [1 + \underline{X}_h' (X'X)^{-1} \underline{X}_h]}$$

"Extra SS" F-test:

$$F^* = \frac{[SSE(\text{reduced}) - SSE(\text{full})] / (\# \text{ parameters in test})}{MSE(\text{full})}$$

$$VIF = \frac{1}{1 - R_j^2}$$

$$AIC = n \ln(SSE_p) - n \ln(n) + 2p$$

$$BIC = n \ln(SSE_p) - n \ln(n) + [\ln(n)]p$$

$$C_p = \frac{SSE_p}{MSE_{\text{full}}} - n + 2p$$

$$MSPR = \frac{\sum_{i=1}^{n^*} (Y_i - \hat{Y}_i)^2}{n^*}$$

$$PRESS = \sum_{i=1}^n (Y_i - \hat{Y}_{i(i)})^2$$

Outliers + Influential Cases:

$$r_i = \frac{e_i}{\sqrt{MSE(1-h_{ii})}}, \quad t_i = \frac{e_i}{\sqrt{MSE_{(i)}(1-h_{ii})}}$$

Rules of Thumb:

$$h_{ii} > \frac{2p}{n}, \quad |DFFITS_i| > 2\sqrt{\frac{p}{n}}, \quad \text{Cook's } D_i > F_{(.5, p, n-p)}$$

$$DFFITS_i = \frac{\hat{Y}_i - \hat{Y}_{i(i)}}{\sqrt{MSE_{(i)} h_{ii}}}, \quad \text{Cook's } D_i = \frac{\sum_{j=1}^n (\hat{Y}_j - \hat{Y}_{j(i)})^2}{p \text{ MSE}}$$