Stat509 Fall 2014 HW2 Solution (without calculation details)
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Instruction: Please finish this homework before the class on 09/23. You will have a quiz based on the homework during the class.

1. Suppose that the random variable Y has the pmf

$$p_Y(y) = \frac{1}{12}(c - 2y), \text{ for } y = 0, 1, 2.$$

   (a) Find the value of c. (*Hint:* $p_Y(0) + p_Y(1) + p_Y(2) = 1$.) Answer: $c = 6$

   (b) Compute expectation and variance of Y. Answer: $E(Y) = 2/3$ and $Var(Y) = 5/9$

2. The range of the random variable X is $\{0, 1, 2, 3, x\}$, where x is unknown. If each value is equally likely and the mean of X is 6, determine x. Answer: $x = 24$

3. A pencil company has four extruders for making pencil lead. The maintenance manager has determined from historical data that the number of extruders to go down (out of operation) on any given day is as follows: 0 extruders, 50%; 1 extruder, 30%; 2 extruders, 10%; 3 extruders, 5%; 4 extruders, 5%.

   (a) Find the probability that 3 or more extruders are down. Answer: 0.1

   (b) What is the expected number of extruders down? $E(X) = 0.85$

   (c) Find the variance for the number of extruders down. $Var(X) = 1.2275$

   (d) Find the standard deviation for the number of extruders down. $\sigma = 1.1079$

4. Airplanes approaching the runway for landing are required to stay within the localizer (a ceratin distance left and right of the runway). When an airplane deviates from the localizer, it is sometimes referred to as an exceedence. Consider one airline at a small airport with six daily arrivals and an exceedence rate of 7%.

   (a) Find the probability that on one day no planes have an exceedence. Answer: 0.647

   (b) Find the probability that at least 1 plane exceeds the localizer. Answer: 0.353

   (c) What is the expected number of planes to exceed the localizer on any given day? Answer: 0.42

   (d) What is the variance for the number of planes to exceed the localizer on any given day? Answer: 0.3906

5. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent.
   (a) What is the probability that the first successful alignment requires exactly four trials? Answer: 0.0064
   (b) What is the probability that the first successful alignment requires at most four trials? Answer: 0.9984
   (c) What is the probability that the first successful alignment requires at least four trials? Answer: 0.008

6. A state runs a lottery in which six numbers are randomly selected from 40 (suppose the order of the numbers is not important), without replacement. A player chooses six numbers before the state's sample is selected.
   (a) What is the probability that the six numbers chosen by a player match all six numbers in the states sample? Answer: 2.6×10^{-7}
   (b) What is the probability that four of the six numbers chosen by a player appear in the states sample? Answer: 0.002
   (c) If a player enters one lottery each week, what is the expected number of weeks until a player matches all six numbers in the state's sample? (hint: consider each week's lottery as a Bernoulli trial) Answer: 3838380

7. A programmer makes 2 errors per 500 lines of code, on average. Suppose Poisson distribution is appropriate to model the number of errors of that programmer.
   (a) What is the probability that the programmer will make 4 or more errors in the next 500 lines of code? Answer: 0.1429
   (b) What is the probability that a programmer makes at least 1 error in the next 250 lines of code? Answer: 0.6321