

Quick Check — Lecture 10

Sampling Distributions and the Central Limit Theorem

Yen-Yi Ho

Department of Statistics, University of South Carolina

Section 1

What a Sampling Distribution Is

Question 1

Setup

We take 500 random samples of $n = 20$ from the same population and record the mean of each one.

The distribution of those 500 sample means is called the ____.

- A.** The population distribution
- B.** The sampling distribution of the sample mean
- C.** The sample distribution of one sample
- D.** The distribution of the parameter

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Answer: B.

Three distributions are in play and they are easy to blur: the **population** (all individuals), a **sample** (the 20 values you actually hold), and the **sampling distribution** (one dot per sample, 500 dots here). **D** is a giveaway — a parameter is a fixed number, so it has no distribution.

Question 2

Setup

Male student heights are approximately normal with $\mu = 69.31$ in and $\sigma = 3.61$ in. We draw random samples of $n = 100$.

What is the standard error of the sample mean?

- A.** 3.61 inches
- B.** 0.36 inches

- C.** 0.81 inches
- D.** 0.04 inches

Question 2

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Answer: B.

$SE(\bar{x}) = \sigma/\sqrt{n} = 3.61/\sqrt{100} = 3.61/10 = 0.36$. **A** forgets to divide at all. **C** is the answer for $n = 20$. **D** divides by n instead of $\sqrt{n}$ — a very common slip, and it makes the sampling distribution look ten times tighter than it is.

Section 2

The Central Limit Theorem

Question 3

Setup

Fish weights across U.S. waters are ****strongly right-skewed****, with $\mu = 320.4$ kg and $\sigma = 316.8$ kg. We take 500 random samples of $n = 100$ fish.

What does the distribution of those 500 sample means look like?

- A.** Strongly right-skewed, like the population
- B.** Approximately normal, centered at 320.4 kg
- C.** Approximately normal, centered at 160.2 kg
- D.** Uniform between 0 and 1500 kg

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Answer: B.

This is the surprising part of the CLT: the sampling distribution does **not** inherit the population's shape. With $n = 100$ the skew washes out and the means pile up symmetrically around the **true mean** $\mu = 320.4$ — not around half of it.

Question 4

Setup

Your sample means are too spread out for the precision you need.

Which change actually SHRINKS the standard deviation of the sampling distribution?

- A. Take 5,000 samples instead of 500
- B. Increase the size n of each sample
- C. Take fewer samples, each unchanged in size
- D. Nothing can change it

Question 4

Setup

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- C. Take fewer samples, each unchanged in size
- D. Nothing can change it

Answer: B.

Only n appears in $\sigma/\sqrt{n}$. The **number of samples** is not in the formula at all — taking more samples just paints a sharper picture of the same distribution. Compare the 500-sample and 5,000-sample columns in the height table: the SDs barely move.

Question 5

Setup

We draw ****ONE**** random sample of size $n = 100$ from an exponential population with mean $\mu = 10$ and sd $\sigma = 10$.

About what value should the standard deviation OF THE SAMPLE be?

- A. About 10
- B. About 1

- C. About 0.1
- D. About 100

Question 5

Setup

We draw **ONE** random sample of size $n = 100$ from an exponential population with mean $\mu = 10$ and sd $\sigma = 10$.

About what value should the standard deviation **OF THE SAMPLE** be?

- A. About 10
- B. About 1

- C. About 0.1
- D. About 100

Answer: A.

Read the capitalized words. The sd **of the sample** estimates $\sigma = 10$, and it does **not** shrink as n grows — a bigger sample just estimates the same spread more reliably. **B** is $\sigma/\sqrt{n}$, which is the sd of the **sample mean**: a different question entirely.

Q	Idea	Answer
1	One dot per sample $\Rightarrow$ sampling distribution	B
2	$SE = \sigma/\sqrt{n}$, not σ/n	B
3	Means go normal even when the population does not	B
4	Only n shrinks the spread; more samples do not	B
5	s estimates σ and does not shrink with n	A

Ask yourself every time: spread of *individuals*, or spread of *means*?