

Quick Check — Lecture 11

Sampling Distributions and Confidence Intervals (Chap 5.2–5.3)

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Section 1

The Sampling Distribution of the Sample Mean

Question 1

Setup

A single random sample of $n = 50$ male students has standard deviation $s = 3.6$ inches. Someone computes

$$\frac{3.6}{\sqrt{50}} = 0.51 \text{ inches.}$$

What is that 0.51 called?

- A.** The standard deviation of the individual heights
- B.** The standard error of the sample mean
- C.** The population standard deviation σ
- D.** The margin of error of a 99% CI

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Answer: B.

Both are “standard deviations,” which is exactly the trap. $s = 3.6$ measures spread among **individual students**. $s/\sqrt{n} = 0.51$ measures spread among **sample means** of size 50 — a quantity we never observe directly, but the CLT hands it to us anyway.

Question 2

Setup

Household incomes in a large city are ****strongly right-skewed****. We take random samples of size n and record $\bar{x}$ each time.

As n gets large, what does the sampling distribution of $\bar{x}$ look like?

- A.** Still strongly right-skewed — it inherits the population's shape
- B.** Approximately normal, centered at μ , with spread $\sigma/\sqrt{n}$
- C.** Approximately normal, centered at the $\bar{x}$ we happened to get
- D.** Approximately normal only if the population is normal

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Answer: B.

Normality of $\bar{x}$ is the CLT's **conclusion**, not its requirement — that is the whole point of **D** being wrong. **A** confuses the population distribution with the sampling distribution. The center is the unknown truth μ , not our one sample.

Section 2

From the CLT to a Confidence Interval

Question 3

Setup

From one sample you compute a 95% CI for the mean class-to-class time:

(3.07, 3.73) minutes.

Which interpretation is correct?

A. There is a 95% probability that μ lies in (3.07, 3.73)

B. 95% of students have times in (3.07, 3.73)

C. In repeated sampling, 95% of intervals built this way contain μ

D. 95% of sample means fall in (3.07, 3.73)

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Answer: C.

A is the one almost everyone picks. But μ is a fixed number and these endpoints are now fixed numbers too — this particular interval either covers μ or it does not. The 95% describes **the procedure**, not this interval. **B** confuses a CI with the spread of individuals.

Question 4

Setup

Same data, same sample, same $\bar{x}$. You simply recompute the interval at ****99%**** confidence instead of 95%.

What happens to the interval?

- A.** Narrower — we are more confident
- B.** Same width; only the label changes
- C.** Wider — we multiply the SE by 2.58 instead of 1.96
- D.** Shifts right, same width

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Answer: C.

More confidence is bought with **less precision**. To be surer of catching μ , you cast a wider net: ± 1.65 SE for 90%, ± 1.96 SE for 95%, ± 2.58 SE for 99%. The only way to get **both** more confidence and a narrower interval is a larger n .

Q	Idea	Answer
1	s spreads individuals; $s/\sqrt{n}$ spreads sample means	B
2	Normality is the CLT's conclusion, not its condition	B
3	The 95% describes the procedure, not this one interval	C
4	More confidence costs width — unless you raise n	C