

Quick Check — Lecture 14

Hypothesis Testing and the CI Duality (Chap 5.4–5.6)

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Section 1

Setting Up the Test

Question 1

Setup

A manufacturer wants the city's sewer-pipe contract. The specification is that the average break point must be **more than** 2400 pplf, and the manufacturer claims its pipes qualify. A sample of $n = 50$ sections gives $\bar{x} = 2460$ pplf with $s = 200$ pplf.

Which setup is correct?

A. $H_0 : \mu > 2400$ vs. $H_a : \mu = 2400$

B. $H_0 : \mu = 2460$ vs. $H_a : \mu \neq 2460$

C. $H_0 : \mu = 2400$ vs. $H_a : \mu > 2400$

D. $H_0 : \mu < 2400$ vs. $H_a : \mu > 2400$

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Answer: C.

The equality always goes under H_0 , and the claim someone must produce evidence *for* goes under H_a . **A** reverses them, which would hand the manufacturer the contract by default. **B** hypothesizes about $\bar{x} = 2460$ — a number we already observed; hypotheses are statements about the unknown parameter. **D** leaves no single value of μ to compute the test statistic under.

Question 2

Setup

We run a two-sided z test at the .05 level of significance.

Which statement describes α correctly?

A. There is a 5% probability that H_0 is true

B. If H_0 is true, we wrongly reject it about 5% of the time

C. There is a 5% probability our decision is wrong

D. 5% of the sample observations fall in the rejection region

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Answer: B.

Watch the conditioning bar: $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true})$. We can compute it precisely *because* it assumes H_0 . **A** and **C** attach a probability to H_0 itself, but μ is a fixed unknown — H_0 is simply true or false. **C** also forgets β : being wrong overall depends on the Type II error rate too, which we usually cannot compute. **D** describes the raw data; the rejection region is a set of values for Z .

Section 2

Reading the Conclusion

Question 3

Setup

A second oceanographer samples $n = 35$ and observes $\bar{x} = 72.9$ fathoms, giving $z_{obs} = 1.41$ and $p = .159$. She **fails to reject** $H_0 : \mu = 72.4$.

What has she shown?

- A. She has proven that $\mu = 72.4$
- B. The probability that H_0 is true is .159
- C. μ is definitely not different from 72.4
- D. The data give too little evidence to conclude $\mu \neq 72.4$

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Answer: D.

Absence of evidence is not evidence of absence — we never *accept* H_0 , we only fail to clear the bar. Note how close this call was: $\bar{x} = 72.9$ sits just short of the cutoff 73.096, so calling it proof of $\mu = 72.4$ would be indefensible. **B** is the classic transposed conditional: p is $P(\text{data this extreme} \mid H_0)$, never $P(H_0 \mid \text{data})$.

Section 3

A Test and an Interval, One Decision

Question 4

Setup

From the same ocean-depth sample ($\bar{x} = 73.2$, $\sigma = 2.1$, $n = 35$), the 95% confidence interval for μ is

$$(72.504, 73.896) \text{ fathoms.}$$

We test $H_0 : \mu = 72.4$ against $H_a : \mu \neq 72.4$ at $\alpha = .05$.

Without computing z , what do you conclude?

- A.** Reject H_0 : 72.4 lies outside the interval
- B.** Fail to reject H_0 : 72.4 is close to 72.504
- C.** Cannot tell — the two answer different questions
- D.** Reject H_0 only once the p -value is reported

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Answer: A.

Both procedures use the same $\bar{x}$, the same $SE = 0.355$, and the same 1.96, so they cannot disagree: the cutoffs sit 0.696 either side of μ_0 , and the interval reaches 0.696 either side of $\bar{x}$.

Question 5

Setup

That test gave $z_{obs} = 2.254$ and $p = .024$, so H_0 was rejected at $\alpha = .05$. Suppose she had set $\alpha = .01$ **before** collecting the data.

What would she conclude?

- A. Reject H_0 — the data have not changed
- B. Reject H_0 — $p = .024$ is still small
- C. Fail to reject: the cutoff moves to ± 2.576 and $.024 > .01$
- D. The test cannot be run at $\alpha = .01$

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- D. The test cannot be run at $\alpha = .01$

Answer: C.

The data have not changed, but the decision rule has — “significant” is always relative to a stated α . Sliding the cutoff out to ± 2.576 shrinks α and grows β . And the duality still holds, between *matching* pieces: the 99% interval $73.2 \pm 2.576(0.355) = (72.286, 74.114)$ does cover 72.4. This is exactly why α is fixed in advance — choose it after seeing $p = .024$ and you can always declare significance.

Q	Idea	Answer
1	The equality goes under H_0 ; the claim goes under H_a	C
2	α is conditional on H_0 being true	B
3	Failing to reject is not proof of H_0	D
4	Same $\bar{x}$, same SE , same $z_{\alpha/2}$ — same verdict	A
5	A smaller α needs a matching wider interval	C

Reject $H_0 \iff$ the matching CI excludes μ_0 .