
Note: please show all of your work to get partial credit. Good luck!
Round probabilities to four decimal places (e.g., 0.0392) and all other numerical answers to two (e.g., 2.12) if needed. Where a question states a different precision, follow that instruction.

1. (10 pts) School district homework survey components.

A school district wants to estimate the average number of minutes per day that students in grades 6–8 spend on homework. It draws a random sample of 900 student ID numbers from the district's enrollment roster as of the first day of the semester and mails a short questionnaire to each selected student's home address; 613 questionnaires are returned.

a) (5 pts) Identify each of the following for this study:

Target population: _____

Sampled population: _____

Sampling unit: _____

Observation unit: _____

Response: _____

b) (5 pts) Explain why the target population and the sampled population may differ in this study.

2. (12pts) Short questions.

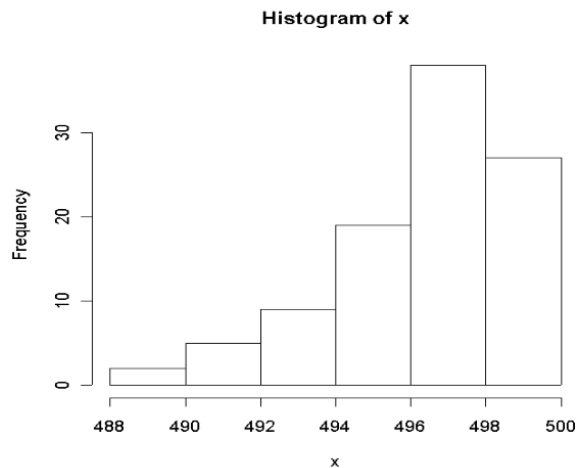
1) (3pts) Suppose there are two populations. The variance of Population 1 is 20, and the variance of Population 2 is 40. Which population shows more variability? Briefly explain?

2) (6pts) (Circle the correct answers) The histogram

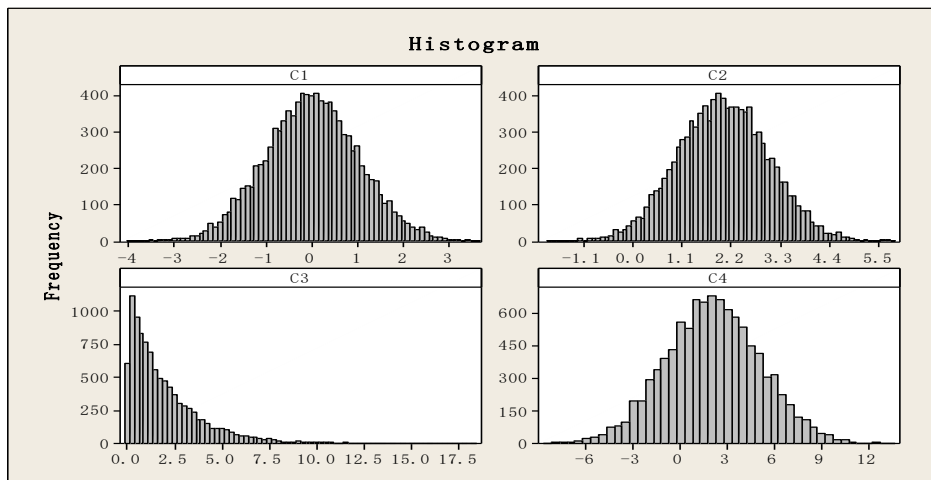
below is skewed left / symmetric / skewed right.

We would expect that its mean would be

larger than / smaller than / equal to its median.



3) (3pts) (Circle the correct answer) Which of the following histograms is the best fit for a normal distribution with mean 2 and standard deviation 1? C1/C2/C3/C4



3. (12pts) Suppose Z is a standard normal random variable.

a.) (2pts) Find z_0 such that $P(Z > z_0) = .017$.

b.) (4pts) Find z_1 such that $P(|Z| > z_1) = .017$. That is, z_1 cuts off a combined area of .017 in the two tails of the distribution.

c.) (6pts) Scores on a placement exam are normally distributed with mean 500 and standard deviation 100. Using your answer from part a.), find the score x_0 that is exceeded by only 1.7% of the students.

4. (6pts) **Selecting a sample.** A random sample of 4 students is to be selected from 30 sociology majors for participation in a special program. In how many different ways can the sample be drawn? And what is the probability that a particular sample is drawn assuming each possible sample has the equal chance being drawn? Leave the probability as an exact fraction.

5. (8pts) **Visually impaired students.** The journal of Visual Impairment & Blindness (May-June 1997) published a study of the lifestyles of visually impaired students. Using diaries, the students kept track of several variables, including number of hours of sleep obtained in a typical day. These visually impaired

students had a mean of 9.06 hours and a standard deviation of 2.11 hours. Assume that the distribution of the number of hours of sleep for this group of students is approximately normal. Find the probability that a visually impaired student gets between 8 and 10 hours of sleep on a typical day.

6.(15pts) Consider the discrete probability distribution of X shown below:

x	10	12	18	20
$p(x)$	.2	.3	.1	.4

a.) Find $E(X)$.

b.) Find $\text{Var}(X)$.

c.) Find $P(X < 15)$.

7.(25pts) **Elderly wheelchair user study.** The American Journal of Public Health (Jan. 2002) reported on a study of elderly wheelchair users who live at home. A sample of 306 wheelchair users, age 65 or older, were surveyed about whether they had an injurious fall during the year and whether their home featured any one of five structural modifications: bathroom modifications, widened

doorways/hallways, kitchen modifications, installed railings, and easy-open doors. The responses are summarized in the table below. Suppose we select, at random, one of the 306 wheel-chair users surveyed.

Home Features	Injurious Fall(s)	No Falls	Totals
All 5	2	7	9
At least 1, but not all	26	162	188
None	20	89	109
Totals	48	258	306

- a.) Find the probability that the wheelchair user had an injurious fall.
- b.) Find the probability that the wheelchair user had all five features installed in the home or had no falls.
- c.) Find the probability that the wheelchair user had no falls and none of the features installed in the home.
- d.) Given no features installed in the home, find the probability of an injurious fall.
- e.) Are “having none features installed in the home” and “having injurious falls” independent? Justify your answer clearly.

8. (12pts) Use of laughing gas. According to the American Dental Association, 60% of all dentists use nitrous oxide (“laughing gas”) in their practice. Let X equal

to number of dentists in a random sample of five dentists who use laughing gas in practice.

a.) Find the expected number of dentists in the sample of five dentists who use laughing gas in practice

b.) What is the standard deviation of the number of dentists in the sample of five dentists who use laughing gas in practice.

c.) Find the probability that exact 2 dentists use laughing gas in the sample of five dentists.

9. (16 points) The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.

a.) What is the probability of no crack requiring repair in a mile of highway?

b.) What is the average number of cracks requiring repair in 5 miles of highway?

c.) What is the probability that at least two cracks require repair in 5 miles of highway?

10. (12 pts) Playing cards. A standard deck of 52 playing cards has 4 suits (Hearts, Diamonds, Clubs, Spades) and 13 ranks (A, 2–10, J, Q, K). A hand of 5 cards is dealt at random.

a.) (3 pts) How many different 5-card hands can be dealt from the deck?

b.) (3 pts) How many 5-card hands contain exactly 3 Aces?

c.) (3 pts) What is the probability that a randomly dealt 5-card hand contains exactly 3 Aces? (Report 4 digits)

d.) (3 pts) How many 5-card hands consist entirely of cards from the same suit (a "flush")?

11. (12 pts) Software quality control. A software company assigns its projects to three development teams. Team A handles 50% of the projects, Team B handles 30%, and Team C handles 20%. The probability that a project contains a critical bug is 0.04 for Team A, 0.06 for Team B, and 0.03 for Team C. One project is selected at random.

a.) (6 pts) Using the Law of Total Probability, find $P(\text{Bug})$, the probability that the selected project contains a critical bug.

b.) (6 pts) The selected project is found to contain a critical bug. Using Bayes' Rule, find the probability that it was developed by Team B.

12. (10 pts) Diagnostic testing. A diagnostic test for a rare disease is 95% sensitive, that is $P(\text{positive} \mid \text{disease}) = 0.95$, and it has a 3% false-positive rate, that is $P(\text{positive} \mid \text{no disease}) = 0.03$. The disease occurs in 1% of the population. A person is selected at random and tested.

a.) (4 pts) Find the probability that the person tests positive.

b.) (4 pts) Using Bayes' formula, find $P(\text{disease} \mid \text{positive test result})$.

c.) (2 pts) Briefly explain why this probability is so much smaller than the 95% sensitivity of the test.