Exam 2 (100pts)
Stat 515 (Fall 2026) **Name:**_____
Instructor: Yen-Yi Ho

Note: please show all of your work to get partial credit. Good luck!
Round probabilities to four decimal places (e.g., 0.0392) and all other numerical answers to two (e.g., 2.12) if needed. Where a question states a different precision, follow that instruction.
This is a 75-minute, closed book, closed note exam. You may use a non-programmable, non-graphing calculator. A formula sheet and standard normal (z) and t tables accompany this exam.

1. (18 pts) **Short questions.**

1) (3 pts) Define what is meant by a Type I error of a test.

2) (3 pts) How is the Type I error of a test related to the significance level (α)?

3) (3 pts) Define what is meant by the p-value of a test.

4) (3 pts) What would happen to the Type II error of a test, if we decrease α?

5) (2 pts) (Circle the correct answers) In general, a $(1-\alpha)100\%$ confidence interval for μ becomes **wider** / **narrower** if the sample size n is increased, and **wider** / **narrower** if the confidence level $(1-\alpha)$ is increased.

6) (2 pts) (Circle the correct answers) A test of hypotheses results in a p-value of 0.0372. If the α-level is 0.01 you would **fail to reject** / **reject** the null hypothesis. If the α-level is 0.05 you would **fail to reject** / **reject** the null hypothesis.

7) (2 pts) (Circle the correct answer) Suppose that for a test of H_0: $\mu = \mu_0$ versus H_a: $\mu \neq \mu_0$ the calculated test statistic is $z = -1.84$. The p-value for this test is **P(Z < −1.84)** / **P(Z > −1.84)** / **2P(Z < −1.84)**

2. (12 pts) **Textbook spending.** A national survey reports that the mean annual amount undergraduates spend on textbooks and course materials is $\mu = \$1{,}240$, with standard deviation $\sigma = \$400$. A random sample of 100 undergraduates is selected, and $\bar{X}$ denotes the sample mean amount spent.

a) (3 pts) Based on the central limit theorem (CLT), find the mean of $\bar{X}$.

b) (3 pts) Based on the CLT, find the standard deviation of $\bar{X}$.

c) (3 pts) Describe the shape of the sampling distribution of $\bar{X}$. (Justify your answer clearly.)

d) (3 pts) How likely is it that the mean amount spent by these 100 students is greater than \$1,290?

3. (10 pts) **Returned orders.** An online retailer knows that 20% of all orders placed on its site are eventually returned. A random sample of 400 orders is selected; let X denote the number of orders in the sample that are returned. Use the normal approximation to the binomial, and do not apply a continuity correction.

a) (4 pts) Find the mean and standard deviation of X, and explain why the normal approximation is appropriate here.

b) (3 pts) Approximate $P(X \geq 92)$.

c) (3 pts) Approximate $P(68 \leq X \leq 92)$.

4. (6 pts) Setting up hypotheses. For each situation, state the appropriate null and alternative hypotheses. Be sure to define any parameter you use in terms of the problem.

a) (3 pts) A tire manufacturer advertises that its all-season tires last, on average, at least 60,000 miles. A consumer group suspects the true mean tread life is less than advertised and plans to test the claim.

b) (3 pts) A school district claims that more than 60% of its graduates enroll in a four-year college. A researcher collects data hoping to find evidence supporting this claim.

5. (10 pts) **Interpreting a confidence interval.** A city utility studied monthly household water use (in gallons). A random sample of 45 households produced a 90% confidence interval of (3,180 gallons, 4,020 gallons).

a) (4 pts) Identify the error in the following interpretation and briefly explain why it is an error: "We are 90% confident that the average monthly water use of the 45 households in the sample is between 3,180 and 4,020 gallons."

b) (4 pts) Identify the error in the following interpretation and briefly explain why it is an error: "There is a 90% probability that the mean monthly water use of all households in the city is between 3,180 and 4,020 gallons."

c) (2 pts) Give a correct interpretation of this interval.

6. (9 pts) Battery charging time. An engineer tests a new fast-charge battery. A random sample of 16 batteries gave a mean charging time of $\bar{x} = 42.5$ minutes with standard deviation $s = 6.0$ minutes.

a) (6 pts) Construct and interpret a 95% confidence interval for the true mean charging time.

b) (3 pts) What must be true about the distribution of charging times in the population for the procedure in part (a) to be valid, and why does that condition matter here in particular?

7. (9 pts) **Sample size determination.**

a) (5 pts) A researcher wants to estimate the mean daily recreational screen time (in hours) of adults to within ±0.25 hours with 95% confidence. A pilot study suggests $\sigma \approx 1.4$ hours. What sample size is needed?

b) (4 pts) A pollster wants to estimate the proportion of registered voters who support a ballot measure to within ±0.03 with 95% confidence. No prior estimate of p is available. What sample size is needed?

8. (14 pts) Knee rehabilitation. Historically, the mean recovery time following a certain knee surgery is 10 weeks. A hospital introduces a new rehabilitation protocol and wants to know whether the mean recovery time under the new protocol differs from 10 weeks. A random sample of 16 patients on the new protocol had a mean recovery time of 8.5 weeks with a standard deviation of 2.4 weeks. Assume recovery times are approximately normally distributed. Use $\alpha = 0.05$.

a) (3 pts) State the null and alternative hypotheses, defining the parameter in terms of the problem.

b) (3 pts) Compute the appropriate test statistic and report its degrees of freedom.

c) (3 pts) Find the rejection region.

d) (3 pts) State your conclusion clearly, in the context of the problem.

e) (2 pts) Using the table, provide the p-value of this test.

9. (12 pts) **Public transit use.** A city transit authority claims that 40% of city residents use public transit at least once a week. In a random sample of 600

residents, 216 reported using public transit at least once a week. Test, at the $\alpha = 0.05$ level, whether the true proportion differs from 0.40.

a) (2 pts) State the null and alternative hypotheses in terms of p, the true proportion of all city residents who use public transit at least once a week.

b) (3 pts) State the conditions required for this test to be valid, verify them, and compute the appropriate test statistic.

c) (2 pts) Find the p-value of the test.

d) (2 pts) State your conclusion clearly, in the context of the problem.

e) (3 pts) Construct a 95% confidence interval for p. Does the interval agree with your test decision in part (d)? Explain.