

--- ANSWER KEY ---

1a) The significance level α is the probability of making a Type I error — rejecting H_0 when it is actually true. It is set by the researcher before the test and defines how much evidence is required to reject H_0 .

1b) A Type I error occurs when we reject H_0 when H_0 is in fact true. The probability of a Type I error equals α .

2) Let $X \sim \text{Binomial}(250, 0.08)$. $\mu = np = 20$, $\sigma = \sqrt{np(1-p)} = \sqrt{18.4} \approx 4.29$.

$P(X > 25) \approx P(Z > (25 - 20)/4.29) = P(Z > 1.17)$ [No continuity correction applied.]

3a) The ordered sample values are plotted against the corresponding quantiles of the standard normal distribution. If the population is normal, the points should fall close to a straight line.

3b) An upward (convex) curve indicates a distribution that is skewed to the right: the upper tail is stretched out relative to a normal distribution, so the largest observations are much larger than normality would predict.

3c) No. The χ^2 -based confidence interval for σ^2 requires normality and is not robust to departures from it — unlike the t procedures for μ , it is not rescued by a large sample size. With clear right skew the interval's actual coverage can be far from the stated level, so the engineer should not use it.

4a) CLT: For a random sample of size n , the sample mean $\bar{x}$ approaches a normal distribution as n increases, regardless of the population shape. n is large enough if: (i) population is normal (always), (ii) $n \geq 30$ for continuous populations, (iii) $np \geq 15$ and $n(1-p) \geq 15$ for binomial.

5a) The population must be normally distributed.

5b) No. The χ^2 CI for variance is NOT robust against non-normality — even mild departures from normality can badly distort the interval.

5c) A procedure is robust against non-normality if it still gives valid results (correct coverage or error rates) when the normality assumption is violated. A procedure that is not robust will produce inaccurate CIs or incorrect p -values as soon as the data depart from normality, even slightly.

6a) Error: The interval is for the population mean, not the sample mean. The sample mean is a fixed number (the point estimate, here $(2104+2896)/2 = 2500$ mg); there is nothing to be "confident" about for it.

6b) Error: Once the data are collected and the interval computed, the population mean μ is a fixed (unknown) constant — not a random variable. The correct interpretation is that 95% of intervals constructed this way in repeated sampling would contain μ ; this particular interval either does or does not.

7) $\hat{p} = 412/980 \approx 0.4204$. 95% CI: $\hat{p} \pm 1.96\sqrt{(\hat{p}(1-\hat{p}))/n} = 0.4204 \pm 1.96\sqrt{(0.4204 \times 0.5796)/980}$
 $= 0.4204 \pm 1.96(0.01576) = 0.4204 \pm 0.0309 \rightarrow (0.3895, 0.4513)$

We are 95% confident that between 38.95% and 45.13% of all undergraduates get fewer than 7 hours of sleep.

8a) $n = (z^* \times \sigma / E)^2 = (1.96 \times 10 / 2)^2 = (9.8)^2 = 96.04 \rightarrow n = 97$

8b) Use $\hat{p} = 0.5$ (most conservative). $n = (z^*/E)^2 \times 0.25 = (1.645/0.04)^2 \times 0.25 = (41.125)^2 \times 0.25 \approx 1691 \times 0.25 \approx 423$

9a) Let μ = the true mean caffeine content (mg) per pod.

$H_0: \mu = 75$ vs. $H_a: \mu < 75$

9b) Let μ_1 = mean length of stay in standard care unit; μ_2 = mean length of stay in enhanced recovery unit.

$H_0: \mu_1 - \mu_2 = 0$ vs. $H_a: \mu_1 - \mu_2 \neq 0$

9c) Let μ = the true mean fill volume (mL) per bottle.

$H_0: \mu = 500$ vs. $H_a: \mu \neq 500$

10a) Let μ = the true mean resting heart rate (bpm) of patients in cardiac rehab.

$H_0: \mu = 72$ vs. $H_a: \mu \neq 72$

10b) $t = (\bar{x} - \mu_0)/(s/\sqrt{n}) = (76.4 - 72)/(9.2/\sqrt{18}) = 4.4/2.169 \approx 2.03$

10c) $df = 17$; two-tailed $\alpha = 0.05 \rightarrow t^* \approx 2.110$. Rejection region: $|t| > 2.110$.

10d) $|t| = 2.03 < 2.110$, so we fail to reject H_0 at $\alpha = 0.05$. There is insufficient evidence at the 5% level to conclude that the mean resting heart rate of cardiac rehab patients differs from 72 bpm.

11a) Fail to reject (p-value $0.0761 > 0.05$).

11b) Reject (p-value $0.0761 < 0.10$).

11c) The two-sided p-value is 0.0761. Because $\bar{x} = 47.3 < 50$, the observed result is in the lower tail. For the one-sided test $H_a: \mu < 50$, the p-value $= 0.0761/2 = 0.03805$.

12a) When $n = 12$ and $X = 1$, $\hat{p} = 1/12 \approx 0.083$. We need $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$ for the normal approximation to be reliable. Here $n\hat{p} \approx 1$, which badly violates the condition — the formula is unreliable.

12b) Wilson's method adds 2 successes and 2 failures (so $\tilde{n} = n+4$, $\tilde{p} = (X+2)/(n+4)$), which pulls $\hat{p}$ away from 0 or 1 and stabilizes the variance estimate. The resulting interval has much better actual coverage, especially for small n or extreme p .