

Quiz 4 — Confidence Intervals for a Mean

STAT 515 · Ott & Longnecker §4.11–4.12, §5.1–5.3 · Covers Homework 6

Name: _____

Date: Wednesday, Oct 21, 2026

Total Points: 40 · Time: 15 minutes

For every confidence interval, write down the point estimate, the standard error, and the multiplier (z^* or t^*) before giving the final interval.

Useful values: $z^* = 1.645$ (90%), 1.96 (95%), 2.576 (99%); $t_{0.025, 15}^* = 2.131$, $t_{0.05, 15}^* = 1.753$.

Problem 1: Interpreting a Confidence Interval (10 points)

A study reports a 95% confidence interval for the mean daily screen time of college students as (4.6 hours, 5.4 hours).

(a) A student writes: “There is a 95% probability that μ is between 4.6 and 5.4 hours.” Explain why this is wrong. Your answer must say something about whether μ is a random quantity. (5 points)

(b) Write one sentence that interprets the interval correctly, and explain what the “95%” actually refers to. (5 points)

Problem 2: Confidence Interval for a Mean, σ Known (12 points)

A machine fills bags of coffee. From extensive past production the population standard deviation is known to be $\sigma = 3.0$ grams. A random sample of $n = 36$ bags has a mean weight of $\bar{x} = 248.5$ grams.

- (a) Compute the standard error of $\bar{X}$. (3 points)

- (b) Construct a 95% confidence interval for the population mean fill weight μ . (5 points)

- (c) Construct a 99% confidence interval for μ . Which interval is wider, and why? (4 points)

Problem 3: Confidence Interval for a Mean, σ Unknown (12 points)

A lab measures the active-ingredient content (in mg) of a random sample of $n = 16$ tablets and obtains

$$\bar{x} = 4.82 \text{ mg}, \quad s = 0.64 \text{ mg}.$$

- (a) Why is a t interval used here rather than a z interval? How many degrees of freedom does it have? (4 points)

- (b) Construct a 95% confidence interval for the mean content μ . (5 points)

(c) The label claims 5.0 mg of active ingredient per tablet. Is the label claim plausible given your interval? Explain. (3 points)

Problem 4: Determining the Sample Size (6 points)

A planner wants to estimate the mean weekly study time of students. From a pilot study, $\sigma \approx 12$ hours.

(a) How many students must be sampled so that a 95% confidence interval has a half-width (E) of at most ± 3 hours? Use $n = \left(\frac{z^* \sigma}{E}\right)^2$ and round correctly. (4 points)

(b) If the planner cuts the margin of error in half, by what factor does the required sample size change? (2 points)