

Quiz 4 — Solutions

STAT 515 · Confidence Intervals for a Mean (Homework 6 material)

Total Points: 40

Problem 1: Interpreting a Confidence Interval (10 points)

(a) (5 points) The population mean μ is a **fixed, unknown constant**, not a random variable. The endpoints 4.6 and 5.4 are also fixed once the sample is drawn. So μ either is in $(4.6, 5.4)$ or it is not — the probability is 0 or 1, and we do not know which. The randomness lies in the **sampling procedure**, not in μ ; before the sample is drawn, the random *interval* has a 95% chance of capturing μ , but after the fact there is no probability left to assign.

Grading: 3 points for stating that μ is fixed, not random; 2 points for locating the randomness in the interval/procedure.

(b) (5 points) Correct interpretation: “We are 95% confident that the true mean daily screen time of all college students is between 4.6 and 5.4 hours.”

The “95%” refers to the long-run **success rate of the method**: if we repeatedly drew random samples of the same size and computed a 95% confidence interval from each, about 95 out of every 100 of those intervals would contain the true μ .

Grading: 2 points for a correct one-sentence interpretation mentioning the population mean; 3 points for the repeated-sampling explanation of the confidence level.

Problem 2: Confidence Interval for a Mean, σ Known (12 points)

$\sigma = 3.0$, $n = 36$, $\bar{x} = 248.5$.

(a) (3 points)

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.0}{\sqrt{36}} = \frac{3.0}{6} = \boxed{0.50 \text{ grams}}$$

(b) (5 points) Point estimate $\bar{x} = 248.5$; multiplier $z^* = 1.96$; standard error 0.50.

$$248.5 \pm 1.96(0.50) = 248.5 \pm 0.98 \implies \boxed{(247.52, 249.48) \text{ grams}}$$

Grading: 1 point point estimate, 1 point $z^* = 1.96$, 1 point margin of error, 2 points for the interval.

(c) (4 points) Multiplier $z^* = 2.576$:

$$248.5 \pm 2.576(0.50) = 248.5 \pm 1.288 \implies \boxed{(247.21, 249.79) \text{ grams}}$$

The **99% interval is wider**. A higher confidence level requires a larger multiplier, so to be more certain of capturing μ we must accept a less precise (wider) interval. Confidence and precision trade off against each other at a fixed sample size.

Grading: 2 points for the interval, 1 point for identifying the wider one, 1 point for the trade-off explanation.

Problem 3: Confidence Interval for a Mean, σ Unknown (12 points)

$n = 16$, $\bar{x} = 4.82$, $s = 0.64$.

(a) (4 points) The population standard deviation σ is **unknown** and is estimated by s . Substituting s for σ adds extra variability, so the standardized quantity follows a t distribution rather than the standard normal. Degrees of freedom: $df = n - 1 = \boxed{15}$.

Grading: 3 points for “ σ unknown, estimated by s ”, 1 point for $df = 15$.

(b) (5 points) Standard error:

$$\frac{s}{\sqrt{n}} = \frac{0.64}{\sqrt{16}} = \frac{0.64}{4} = 0.16$$

Multiplier $t_{0.025, 15}^* = 2.131$:

$$4.82 \pm 2.131(0.16) = 4.82 \pm 0.341 \implies \boxed{(4.479, 5.161) \text{ mg}}$$

Grading: 1 point standard error, 1 point $t^* = 2.131$, 1 point margin of error, 2 points interval.

Common error: using $z^* = 1.96$ — deduct 2 points.

(c) (3 points) Yes. The claimed value $\mu = 5.0$ mg **lies inside** the 95% interval $(4.479, 5.161)$, so 5.0 mg is a plausible value for the mean content; these data give no evidence against the label claim.

Grading: 1 point for “plausible / yes”, 2 points for the reason (5.0 is inside the interval).

Problem 4: Determining the Sample Size (6 points)

(a) (4 points)

$$n = \left(\frac{z^* \sigma}{E} \right)^2 = \left(\frac{1.96 \times 12}{3} \right)^2 = (7.84)^2 = 61.47 \implies \boxed{n = 62}$$

Always round **up**: $n = 61$ would give a margin of error slightly larger than 3 hours, failing the requirement.

Grading: 1 point formula, 2 points value 61.47, 1 point for rounding up to 62.

(b) (2 points) Halving E **quadruples** the required sample size, because n is inversely proportional to E^2 . (Check: $E = 1.5$ gives $n = 245.86 \rightarrow 246 \approx 4 \times 62$.)

Grading: 1 point for “factor of 4”, 1 point for the E^2 reasoning.

R verification

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# Problem 2
se <- 3 / sqrt(36)
248.5 + c(-1, 1) * qnorm(0.975) * se # 247.52 249.48
248.5 + c(-1, 1) * qnorm(0.995) * se # 247.21 249.79

# Problem 3
se <- 0.64 / sqrt(16)
4.82 + c(-1, 1) * qt(0.975, df = 15) * se # 4.479 5.161

# Problem 4
(qnorm(0.975) * 12 / 3)^2 # 61.47 -> 62
(qnorm(0.975) * 12 / 1.5)^2 # 245.86 -> 246
```