Quiz 5 — Hypothesis Testing for a Mean

STAT 515 · Ott & Longnecker §5.4–5.8 · Covers Homework 7

Name: _____ **Date:** Wednesday, Oct 28, 2026

Total Points: 40 · Time: 15 minutes

For the test in Problem 3, state the hypotheses, the test statistic, your decision, and a conclusion **in the context of the problem**. A decision without a contextual conclusion earns only partial credit.

Useful values: $z_{0.025} = 1.96$, $z_{0.05} = 1.645$.

Problem 1: Setting Up Hypotheses (8 points)

For each scenario, define the parameter of interest **in words and symbols**, state H_0 and H_a, and say whether the test is one-sided or two-sided. Do not carry out any test.

(a) A cereal box label claims the box contains **at least** 500 grams. A consumer group suspects boxes contain less than the label claims. (4 points)

(b) A clinic's historical mean wait time is 22 minutes. After a scheduling change, administrators want to know whether the mean wait time has **changed**. (4 points)

Problem 2: Types of Error (10 points)

A machine is supposed to fill bottles to 500 mL. The plant manager tests $H_0 : \mu = 500$ against $H_a : \mu \neq 500$.

(a) Describe what a **Type I error** would mean in this specific setting. (3 points)

(b) Describe what a **Type II error** would mean in this specific setting. (3 points)

(c) If the manager lowers α from 0.05 to 0.01 with the sample size unchanged, what happens to the probability of a Type I error, and what happens to the probability of a Type II error? (4 points)

Problem 3: Large-Sample z-Test (16 points)

A psychologist studies simple reaction time. For the general adult population the mean reaction time is 250 milliseconds. A random sample of $n = 36$ athletes gives

$$\bar{x} = 242.0 \text{ ms}, \qquad s = 24.0 \text{ ms}.$$

The psychologist wants to know whether the mean reaction time of athletes **differs** from 250 ms.

(a) State H_0 and H_a. (3 points)

(b) Explain why a z test is appropriate here, then compute the standard error and the test statistic $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$. (4 points)

(c) The two-sided p-value is 0.0455. Using $\alpha = 0.05$, state your decision and write a conclusion **in context**. (4 points)

(d) Construct a 95% confidence interval for μ. (3 points)

(e) Explain how the interval in (d) is consistent with your decision in (c). Be specific about what feature of the interval corresponds to your decision. (2 points)

Problem 4: Interpreting a p-Value (6 points)

(a) Write a sentence defining what a p-value of 0.0455 means. Your sentence must include the phrase "assuming H_0 is true." (3 points)

(b) True or false, with a one-sentence justification: *"A p-value of 0.0455 means there is a 4.55% probability that* H_0 is true." (3 points)