STAT 515 — Homework 10

Due: Wednesday, December 2, 2026

Submit a **hard copy** at the start of class. Email your R code (a single `.R` file) to Alex Goebel before class. Label each part clearly and show all work for full credit.

This assignment covers one-way analysis of variance (§8.2–8.4, §9.5) from Lecture 18.

Useful values: $F_{.05,\,2,\,21} = 3.467$; $F_{.01,\,2,\,21} = 5.780$; $F_{.05,\,3,\,20} = 3.098$; $F_{.01,\,3,\,20} = 4.938$; $q_{.05,\,3,\,21} = 3.565$.

Problem 1 (15 pts) — Study Methods and Exam Scores

A psychology instructor randomly assigns 24 students to one of three study methods — **Method A** (practice problems), **Method B** (spaced repetition), or **Method C** (re-reading) — with 8 students per group. The students take a standardized exam at the end of the unit. Their scores are:

Method A	72	74	76	78	70	75	73	78
Method B	85	82	88	80	86	83	87	81
Method C	70	74	73	76	68	71	74	76

(a) (3 pts) Compute the three group sample means $\bar{y}_A$, $\bar{y}_B$, $\bar{y}_C$ and the grand mean $\bar{y}$.

(b) (5 pts) Compute SST, SSE, and TSS. Verify that SST + SSE = TSS. Then fill in the ANOVA table below.

Source	SS	df	MS	F
Treatments				
Error				
Total				

(c) (4 pts) State H_0 and H_a, give the rejection region at $\alpha = 0.05$, and state your conclusion in the context of the problem.

(d) (3 pts) Which assumption of one-way ANOVA can you check from the sample standard deviations alone? Check it, then name one assumption that cannot be checked from the data summary.

Problem 2 (12 pts) — Completing an ANOVA Table

A one-way ANOVA is run on $k = 4$ groups with $n_i = 6$ observations each ($n = 24$). A partial ANOVA table is given below.

Source	SS	df	MS	F
Treatments	90			
Error	180			
Total				

(a) (4 pts) Fill in all missing entries (df, MS, F, and TSS).

(b) (4 pts) Using the critical values provided, can you reject H_0 at $\alpha = 0.05$? At $\alpha = 0.01$? Give the p-value bracket and explain.

(c) (4 pts) Suppose instead the same total sum of squares (TSS = 270) arose with SST = 150 and SSE = 120. Recompute F. How does the conclusion change? What does this tell you about which component of variation matters for the test?

Problem 3 (10 pts) — Why One-Way ANOVA?

(a) (4 pts) A researcher has $k = 5$ groups and plans to compare all pairs of group means using individual two-sample t-tests at $\alpha = 0.05$.

(i) How many pairwise t-tests would be required?

(ii) If the null hypothesis is true for every pair, what is the probability that **at least one** test incorrectly rejects? (Express your answer as a decimal rounded to four places.)

(b) (4 pts) Explain in two or three sentences why running a single one-way ANOVA F-test is preferable to running all the pairwise t-tests in part (a). What error rate does the ANOVA control?

(c) (2 pts) The F-test for one-way ANOVA is always **upper-tailed**. Briefly explain why a lower-tailed or two-tailed F-test for this purpose makes no sense.

Problem 4 (10 pts) — Tukey's Comparisons for Problem 1

Return to the data from Problem 1.

(a) (4 pts) Compute Tukey's HSD at the family-wise $\alpha = 0.05$ level. Show the formula and each component clearly.

$$\text{HSD} = q_{\alpha,\, k,\, n-k} \sqrt{\frac{\text{MSE}}{n_i}}$$

(b) (4 pts) Compare all three pairs of group means to the HSD. State which differences are statistically significant.

Pair	$\lvert\bar{y}_i - \bar{y}_j\rvert$	Significant?
A vs. B		
A vs. C		
B vs. C		

(c) (2 pts) Draw the **underline display** (connecting-letters diagram) for the three study methods, ordered from smallest to largest mean.

Problem 5 (8 pts) — R Analysis

Use R to reproduce the ANOVA and Tukey comparisons for the Problem 1 data. Email your `.R` file to Alex Goebel. Include the following in your submission:

(a) (3 pts) Run `aov()` and `summary()` to produce the ANOVA table. Paste the R output (copied from the console) and confirm that your hand-computed F statistic from Problem 1 matches the R output to two decimal places.

(b) (3 pts) Run `TukeyHSD()` on the fitted model. Paste the output. Which pairs are listed as significant at $\alpha = 0.05$ (adjusted $p < 0.05$)? Do these match your answer in Problem 4(b)?

(c) (2 pts) Use `plot(TukeyHSD(...))` to produce the confidence-interval plot for all pairwise differences. Describe in one sentence what the plot shows, and identify which interval does **not** exclude zero.