

Homework 2 — Probability, Counting, Bayes' Formula (§4.1–4.5)

(Due: Wednesday Sep 09, before class)

Please submit a hard copy of your homework before class and email your R code, if applicable, to Alex Goebel at AJGOEBEL@email.sc.edu

1. A statistics class has 8 juniors and 12 seniors. A committee of 4 students is selected at random. (a) In how many ways can the committee be formed? (b) What is the probability the committee has exactly 2 juniors and 2 seniors?

Solution: (a) $C(20,4) = 4845$. (b) $\text{Favorable} = C(8,2) \cdot C(12,2) = 28 \cdot 66 = 1848$. $P = 1848/4845 = 0.3814$.

2. Let A and B be events with $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$. (a) Find $P(A \cup B)$. (b) Find $P(A | B)$. (c) Are A and B independent? Justify using the definition of independence.

Solution: (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$. (b) $P(A|B) = P(A \cap B)/P(B) = 0.2/0.5 = 0.4$. (c) Since $P(A|B) = 0.4 = P(A)$, A and B ARE independent (equivalently $P(A \cap B) = P(A)P(B) = 0.4 \times 0.5 = 0.20$, which matches).

3. A diagnostic test for a rare disease is 95% sensitive ($P(\text{positive} | \text{disease}) = 0.95$) and has a 3% false-positive rate ($P(\text{positive} | \text{no disease}) = 0.03$). The disease occurs in 1% of the population. Use Bayes' formula to find $P(\text{disease} | \text{positive test result})$.

Solution: $P(\text{pos}) = P(\text{pos}|D)P(D) + P(\text{pos}|D^c)P(D^c) = (0.95)(0.01) + (0.03)(0.99) = 0.0392$. $P(D|\text{pos}) = (0.95)(0.01)/0.0392 = 0.2423 \approx 24.2\%$. Even with a good test, most positives are false positives because the disease is rare.

4. A password must consist of 3 distinct letters followed by 2 distinct digits (0–9). How many different passwords are possible?

Solution: Letters: $P(26,3) = 26 \cdot 25 \cdot 24 = 15600$. Digits: $P(10,2) = 10 \cdot 9 = 90$. Total = $15600 \times 90 = 1,404,000$.