Homework 3 — Probability & Counting Techniques (§4.2–4.5)
Due: Wednesday Sep 16, 2026 before class

Total: 85

Name:

Topics covered: Sample spaces, events, probability rules, conditional probability, independence, total probability rule, Bayes' Rule.

Problem 1: Die and Coin Experiment (20 points)

A fair six-sided die is rolled and a fair coin is flipped simultaneously.

(a) List all elements of the sample space S. (How many outcomes are there total?)

(b) Let A = "die shows an even number" and B = "coin shows Heads." List the elements of A, B, $A \cap B$, and $A \cup B$.

(c) Find P(A), P(B), $P(A \cap B)$, and $P(A \cup B)$. Are events A and B mutually exclusive? Are they independent? Justify your answers.

(d) Let C = "die shows a number greater than 4." Find $P(A \mid C)$ and determine whether A and C are independent.

Problem 2: Student Health Survey (20 points)

A university health survey of 200 students found:

- 120 students exercise regularly (event E)
- 90 students eat a balanced diet (event D)
- 60 students do both (exercise regularly AND eat a balanced diet)

(a) Construct a 2×2 contingency table showing the joint frequencies for E and D.

(b) Find $P(E \cup D)$ using the Addition Rule. Interpret the result in context.

(c) Find $P(E \mid D)$. Interpret this conditional probability.

(d) Are events E and D independent? Show your work using the multiplication rule for independence.

Problem 3: Software Quality Control (10 points)

A software company has three development teams: Team A (50% of projects), Team B (30% of projects), and Team C (20% of projects). The probabilities that a project contains a critical bug are 0.04, 0.06, and 0.03 for Teams A, B, and C, respectively.

(a) Using the Total Probability Rule, find P(Bug), the probability that a randomly selected project has a critical bug.

(b) A project is found to have a critical bug. Using Bayes' Rule, find the probability that it was developed by Team B.

Problem 4: Fundamental Theorem of Counting (15 points)

A computer password is created under the following rules:

- The first character must be an uppercase letter (A–Z): 26 choices
- The next three characters must each be a digit (0–9): 10 choices each
- The last two characters must each be a lowercase letter (a–z): 26 choices each

(a) Using the Fundamental Theorem of Counting, determine the total number of valid passwords.

(b) Suppose repetition of digits is not allowed (the three digits must all be different). How many valid passwords exist?

(c) If a password is chosen uniformly at random from all valid passwords (with repetition allowed), what is the probability that the password begins with the letter "A" and ends with "z"?

Problem 5: Combinations (20 points)

A standard deck of 52 playing cards is used for the following experiments.

- 52 cards total: 4 suits (Hearts, Diamonds, Clubs, Spades) × 13 ranks (A, 2–10, J, Q, K)
- C(N, r) = N! / [r! (N−r)!] = number of ways to choose r items from N without order

(a) How many different 5-card hands can be dealt from a standard 52-card deck?

(b) How many 5-card hands contain exactly 3 Aces?

(c) What is the probability that a randomly dealt 5-card hand contains exactly 3 Aces? Express your answer as a fraction and as a decimal.

(d) How many 5-card hands consist entirely of cards from the same suit (a "flush")? (Include the special cases: straight flush and royal flush are still flushes.)