Homework 4 — Binomial, Poisson, Continuous Random Variables (§4.6–4.9)

(Due: Wednesday Sep 23, before class)

Please submit a hard copy of your homework before class and email your R code, if applicable, to Alex Goebel at AJGOEBEL@email.sc.edu

1. A multiple-choice quiz has 10 independent questions; each has a 30% chance of being guessed correctly ($p = 0.3$). Let X = number of correct guesses. (a) Find $P(X = 4)$. (b) Find $P(X \geq 2)$. (c) Find $E(X)$ and $Var(X)$.

2. Customers arrive at a help desk at a rate of $\lambda = 4$ per 10-minute interval, following a Poisson process. (a) Find P(exactly 3 arrivals in 10 minutes). (b) Find P(at most 2 arrivals in 10 minutes). (c) What are the mean and variance of the number of arrivals?

3. A continuous random variable X has probability density function $f(x) = 2x$ for $0 \leq x \leq 1$, and 0 elsewhere. (a) Verify that $f(x)$ is a valid density. (b) Find $P(0.25 \leq X \leq 0.75)$. (c) Find $E(X)$.

4. An insurance office finds that 8% of claims filed are fraudulent. In a random sample of 15 claims, use the binomial distribution to find the probability that at most 1 claim is fraudulent.

5. Check your answer in Question 4 with R.

6. In R, `dpois(2, lambda=5)` and `ppois(2, lambda=5)` are computed. Explain in words what each command returns, then compute both by hand and check they are consistent (i.e., ppois should be the running total that includes dpois(0), dpois(1), dpois(2)).

Problem 7: Normal Distribution and the Empirical Rule (15 points)

The Normal distribution (Gaussian) is the most important continuous distribution in statistics. It is described by two parameters: the mean μ and the standard deviation σ.

- Bell-shaped and symmetric about μ
- Mean = Median = Mode = μ

- Support: $-\infty < x < +\infty$
- Notation: $X \sim N(\mu, \sigma^2)$

Empirical Rule (68–95–99.7 Rule):
- 68% of data falls within 1 standard deviation of the mean: $(\mu - \sigma, \mu + \sigma)$
- 95% of data falls within 2 standard deviations: $(\mu - 2\sigma, \mu + 2\sigma)$
- 99.7% of data falls within 3 standard deviations: $(\mu - 3\sigma, \mu + 3\sigma)$

The heights of adult men in a large population follow a Normal distribution with mean $\mu = 70$ inches and standard deviation $\sigma = 3$ inches: $X \sim N(70, 9)$.

(a) Sketch (or describe) the shape of this Normal distribution. Label the mean and the points one, two, and three standard deviations above and below the mean.

(b) Using the Empirical Rule, find the probability that a randomly selected man has a height between 67 and 73 inches. Show which interval of the Empirical Rule you are using.

(c) Using the Empirical Rule, find $P(64 < X < 76)$. Interpret this probability in context.

(d) Using the Empirical Rule and symmetry, find $P(X > 76)$. Show your reasoning step by step.

(e) A height of 61 inches is how many standard deviations below the mean? Is a height of 61 inches considered unusual? Justify your answer using the Empirical Rule.