

Homework 5 — Normal Distributions, Q–Q Plots, the Exponential Distribution, and Sampling Distributions

STAT 515 · Chapters 4.9–4.13 (Lectures 8–10)

(Due: Wednesday, Oct 14, 2026, before class)

Please submit a hard copy of your homework before class and email your R code, if applicable, to Alex Goebel at AJGOEBEL@email.sc.edu.

Show your work. For problems that ask for a probability, write down the z -score (or the exponential formula) you used before giving the final number. You may use the standard normal table or R.

Problem 1: Assessing Normality and the Normal Q–Q Plot (20 points)

A campus transportation study recorded the one-way commute time (in minutes) for a random sample of $n = 10$ students:

18, 21, 23, 24, 26, 26, 28, 30, 33, 37

For a sample of size $n = 10$, the theoretical standard normal quantiles $q_i = \Phi^{-1}\left(\frac{i-0.5}{n}\right)$ are:

i	1	2	3	4	5	6	7	8	9	10
q_i	−1.645	−1.036	−0.674	−0.385	−0.126	0.126	0.385	0.674	1.036	1.645

- (a) Compute the sample mean $\bar{x}$ and the sample standard deviation s . (Round to two decimals.)
- (b) Compute the intervals $\bar{x} \pm s$ and $\bar{x} \pm 2s$, and report the percentage of the 10 observations that fall in each. Compare these percentages to the Empirical Rule (68% and 95%).
- (c) Find Q_1 , Q_3 , and the IQR, and compute the ratio IQR/s . How close is this ratio to 1.35, the value expected for normal data?
- (d) Sort the data and pair each ordered value $x_{(i)}$ with its quantile q_i from the table above. Sketch the normal Q–Q plot (put q_i on the horizontal axis and $x_{(i)}$ on the vertical axis) and draw an approximate straight reference line through the points.
- (e) Based on parts (b), (c), and (d) together, is it reasonable to treat these commute times as coming from a normal distribution? Explain in one or two sentences.
- (f) Verify your answers to (a)–(d) in R. Report the output of the following commands:

```
commute <- c(18, 21, 23, 24, 26, 26, 28, 30, 33, 37)
mean(commute)
sd(commute)
quantile(commute, c(0.25, 0.75), type = 6)
IQR(commute, type = 6) / sd(commute)
qqnorm(commute); qqline(commute)
```

Problem 2: Normal Approximation to the Binomial (15 points)

In a large city, 20% of adults commute to work using public transit. A random sample of $n = 900$ adults is selected. Let X be the number in the sample who use public transit, so $X \sim \text{binomial}(n = 900, p = 0.20)$.

- (a) Find $\mu = np$ and $\sigma = \sqrt{np(1-p)}$ for X . Explain why a normal approximation is appropriate here.
 - (b) Using the normal approximation, find $P(X \leq 165)$.
 - (c) Using the normal approximation, find $P(165 \leq X \leq 195)$.
 - (d) Check your answers to (b) and (c) against the exact binomial values in R using `pbinom()`. How close is the approximation?
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Problem 3: The Exponential Distribution (15 points)

Patients arrive at a campus health center according to a Poisson process at a rate of $\lambda = 3$ patients per hour. Let Y be the waiting time (in hours) until the next patient arrives.

- (a) State the distribution of Y , including the value of the parameter θ . What are the mean and standard deviation of Y , in **minutes**?
 - (b) Find the probability that the next patient arrives more than 30 minutes from now. Use $P(Y \geq a) = e^{-a/\theta}$.
 - (c) Find the probability that the next patient arrives within the next 10 minutes.
 - (d) Find the value a (in minutes) such that 90% of waiting times are shorter than a . (That is, find the 90th percentile of Y .)
 - (e) In one sentence, explain the relationship between the Poisson distribution used to count arrivals and the exponential distribution used for the time between arrivals.
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Problem 4: Sums of Independent Normals vs. the Sample Mean (15 points)

A coffee roaster fills bags whose weights are independent and normally distributed with mean $\mu = 340$ grams and standard deviation $\sigma = 4$ grams. A case contains 12 bags.

- (a) Let $T = X_1 + \cdots + X_{12}$ be the total weight of a case. State the distribution of T , giving its mean and standard deviation.
 - (b) Find the probability that a case weighs less than 4,060 grams.
 - (c) Let $\bar{X}$ be the **mean** weight of the 12 bags in a case. State the distribution of $\bar{X}$ and compute its standard error $\sigma_{\bar{X}} = \sigma/\sqrt{n}$.
 - (d) Find $P(\bar{X} > 342)$.
 - (e) Explain why $\sigma_{\bar{X}} = 1.15$ grams is so much smaller than $\sigma = 4$ grams, even though both describe variability in bag weight.
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Problem 5: The Central Limit Theorem (15 points)

Service times at a drive-through window are strongly **right-skewed**, with population mean $\mu = 4.5$ minutes and population standard deviation $\sigma = 2.4$ minutes. A random sample of $n = 36$ customers is taken.

- (a) Can we say the sampling distribution of $\bar{X}$ is *exactly* normal? Approximately normal? Justify your answer by stating the Central Limit Theorem and checking its condition.
 - (b) Find the mean and the standard error of the sampling distribution of $\bar{X}$.
 - (c) Find $P(\bar{X} > 5.0)$.
 - (d) Find $P(4.2 < \bar{X} < 5.0)$.
 - (e) Suppose the sample size is increased from $n = 36$ to $n = 144$. Compute the new standard error. By what factor did the standard error change, and by what factor did the sample size change? State the general relationship between the two.
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Problem 6: Sampling Distribution of a Sample Proportion (10 points)

Suppose 60% of all students at a large university own a car ($p = 0.60$). A random sample of $n = 400$ students is selected, and $\hat{p}$ denotes the sample proportion who own a car.

- (a) Verify the large-sample rule ($n\hat{p} \geq 15$ and $n(1 - \hat{p}) \geq 15$), then state the approximate sampling distribution of $\hat{p}$, including its mean and standard deviation $\sqrt{pq/n}$.
- (b) Find $P(\hat{p} > 0.65)$.
- (c) Find $P(0.57 < \hat{p} < 0.63)$.