

Homework 6 — The CLT in Practice and Confidence Intervals for a Mean

STAT 515 · Ott & Longnecker §4.11–4.12, §5.1–5.3

(Due: Wednesday, Oct 21, 2026, before class)

Please submit a hard copy of your homework before class and email your R code, if applicable, to Alex Goebel at AJGOEBEL@email.sc.edu.

Show your work. For every confidence interval, write down the point estimate, the standard error, and the multiplier (z^* or t^*) before giving the final interval. You may use the tables or R.

Useful values: $z^* = 1.645$ (90%), 1.96 (95%), 2.576 (99%); $t_{0.025, 11}^* = 2.201$, $t_{0.05, 11}^* = 1.796$.

Problem 1: Conditions for the Central Limit Theorem (12 points)

(a) State the Central Limit Theorem. In your statement, be explicit about (i) what is assumed about the population, (ii) what is assumed about the sample, and (iii) what conclusion follows about the sampling distribution of $\bar{X}$.

(b) For each study below, say whether the sampling distribution of $\bar{X}$ can be treated as approximately normal, and give a one-sentence reason. If a condition fails, name which one.

1. A researcher wants the mean daily commute time of students. She selects 10 students at random and records each student's commute on 4 different days, giving 40 measurements, which she treats as a sample of size $n = 40$.
2. A random sample of $n = 12$ claims is drawn from a strongly right-skewed population of insurance claim amounts.
3. A random sample of $n = 200$ claims is drawn from that same right-skewed population.
4. A random sample of $n = 50$ employees is drawn without replacement from a company with 300 employees.

(c) In part (b1), explain in one or two sentences why the 40 measurements are not 40 independent observations, and what effect this has on the true standard error compared with $\sigma/\sqrt{40}$.

Problem 2: Confidence Interval for a Mean, σ Known (15 points)

A manufacturer tests the run time of a rechargeable battery. From extensive past production the population standard deviation is known to be $\sigma = 1.2$ hours. A random sample of $n = 36$ batteries has a mean run time of $\bar{x} = 14.8$ hours.

- (a) Compute the standard error of $\bar{X}$.
 - (b) Construct a 95% confidence interval for the population mean run time μ .
 - (c) Construct a 99% confidence interval for μ .
 - (d) Which interval is wider, and why? Explain the trade-off between confidence level and interval width in one or two sentences.
 - (e) The engineer says, “Because σ is known, I do not need to worry about the shape of the population.” Is she right? Explain.
-

Problem 3: Interpreting a Confidence Interval (15 points)

A nutrition study reports a 95% confidence interval for the mean daily sodium intake of U.S. adults as

(3,210 mg, 3,480 mg).

- (a) A news article writes: “95% of U.S. adults consume between 3,210 and 3,480 mg of sodium per day.” Explain why this is wrong.
 - (b) A student writes: “There is a 95% probability that μ lies between 3,210 and 3,480 mg.” Explain why this is wrong. Your answer should say something about whether μ is a random quantity.
 - (c) Write one sentence that interprets the interval correctly.
 - (d) Explain what the “95%” actually refers to. If 100 different random samples were taken and a 95% CI computed from each, roughly how many of those intervals would contain μ ?
 - (e) The current public-health guideline is 2,300 mg per day. Based on this interval, what can you conclude about whether the mean intake meets the guideline? Explain.
-

Problem 4: Confidence Interval for a Mean, σ Unknown (15 points)

A quality lab measures the protein content (in grams) of 12 randomly selected energy bars:

20.1, 19.6, 21.3, 20.8, 19.9, 20.5, 21.0, 20.2, 20.4, 20.7, 20.9, 20.6

- (a) Compute $\bar{x}$ and s . (You may verify with R, but show the setup.)
- (b) Why is a t interval used here rather than a z interval? How many degrees of freedom does it have?
- (c) Construct a 95% confidence interval for the mean protein content μ .
- (d) Construct a 90% confidence interval for μ , and compare its width to the interval in (c).
- (e) The label claims 20 grams of protein per bar. Is the label claim plausible given your 95% interval? Explain.

(f) What condition must the data satisfy for this interval to be valid, and how would you check it with the 12 observations you have?

Problem 5: Determining the Sample Size for a Mean (10 points)

A transportation planner wants to estimate the mean one-way commute time of students at a large university. From a pilot study, $\sigma \approx 7$ minutes.

(a) How many students must be sampled so that a 95% confidence interval has a margin of error of at most ± 2 minutes? Show the formula $n = \left(\frac{z^* \sigma}{E} \right)^2$ and round correctly.

(b) Repeat for a margin of error of at most ± 1 minute.

(c) The required sample size in (b) is about how many times the one in (a)? State the general rule relating the margin of error to the sample size.

(d) Explain in one sentence why the sample size formula always rounds *up*, never to the nearest integer.

Problem 6: Confidence Intervals in R (8 points)

(a) Verify your answer to Problem 4(c) with the following R code and report the output:

```
bar <- c(20.1, 19.6, 21.3, 20.8, 19.9, 20.5,  
        21.0, 20.2, 20.4, 20.7, 20.9, 20.6)  
mean(bar); sd(bar)  
t.test(bar)$conf.int
```

(b) Modify one argument of `t.test()` to produce the 90% interval from Problem 4(d), and report the result.

(c) For Problem 2, R has no built-in command when σ is known. Write two or three lines of R that reproduce your 95% interval from Problem 2(b) using `qnorm()`.