

Homework 7 — Hypothesis Testing for a Mean

STAT 515 · Ott & Longnecker §5.4–5.8

(Due: Wednesday, Oct 28, 2026, before class)

Please submit a hard copy of your homework before class and email your R code, if applicable, to Alex Goebel at AJGOEBEL@email.sc.edu.

For every test, state the hypotheses, the test statistic with its degrees of freedom, the p-value, your decision, and a conclusion **in the context of the problem**. A decision without a contextual conclusion earns only partial credit.

Useful values: $t_{0.025, 17}^* = 2.110$, $t_{0.05, 17}^* = 1.740$.

Problem 1: Setting Up Hypotheses (15 points)

For each scenario: define the parameter of interest **in words and symbols**, state H_0 and H_a , and say whether the test is one-sided or two-sided. Do not carry out any test.

- (a) A beverage label claims the drink contains **at most** 80 mg of caffeine per can. A consumer group suspects cans contain more than the label claims and collects a random sample of cans.
 - (b) At a hospital, the historical mean length of stay after a certain surgery is 4.5 days. A new recovery protocol is introduced, and administrators want to know whether the mean length of stay has **changed**.
 - (c) A machine is supposed to fill bottles to 500 mL. The plant manager wants to detect whether the machine has drifted out of calibration **in either direction**.
 - (d) In one or two sentences, explain why the null hypothesis always contains the equality, and why the claim a researcher hopes to demonstrate is usually placed in H_a .
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Problem 2: Significance Level and Types of Error (12 points)

- (a) Define the significance level α in one sentence.
- (b) Define a **Type I error** in general terms, then describe what a Type I error would mean specifically in the fill-machine setting of Problem 1(c).
- (c) Define a **Type II error**, and describe what it would mean in the same fill-machine setting.

(d) Which error is more costly for the plant manager? Defend your answer briefly — there is no single correct choice, but your reasoning must be consistent.

(e) If the manager lowers α from 0.05 to 0.01 with the sample size unchanged, what happens to the probability of a Type I error, and what happens to the probability of a Type II error? Explain the trade-off.

Problem 3: Interpreting a p-Value (12 points)

A two-sided test of $H_0: \mu = 100$ against $H_a: \mu \neq 100$ produces a p-value of 0.068.

(a) Write a sentence defining what this p-value means. Your sentence must mention the phrase “assuming H_0 is true.”

(b) What is the decision at $\alpha = 0.05$? State the conclusion in words.

(c) What is the decision at $\alpha = 0.10$? State the conclusion in words.

(d) Parts (b) and (c) reach different decisions from the same data. Explain how that is possible and what it tells you about reporting a p-value rather than only “reject” or “fail to reject.”

(e) Suppose the sample mean fell **above** 100 and the researcher had planned a one-sided test of $H_a: \mu > 100$ from the start. What would the one-sided p-value be?

(f) True or false, with a one-sentence justification: * “A p-value of 0.068 means there is a 6.8% probability that* H_0 is true.”

Problem 4: One-Sample t -Test (18 points)

A cardiologist studies the resting heart rate of patients enrolled in a cardiac rehabilitation program. For healthy adults of the same age, the mean resting heart rate is 78 beats per minute. A random sample of $n = 18$ rehab patients gives

$$\bar{x} = 74.2 \text{ bpm}, \quad s = 8.6 \text{ bpm}.$$

The cardiologist wants to know whether the mean resting heart rate of rehab patients **differs** from 78 bpm.

(a) State H_0 and H_a .

(b) Compute the standard error and the test statistic $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$. Report the degrees of freedom.

(c) The two-sided p-value is 0.078. Using $\alpha = 0.05$, state your decision and write a conclusion in context.

(d) Would your decision change at $\alpha = 0.10$? State the new conclusion in context.

(e) Construct a 95% confidence interval for μ .

(f) Explain how the interval in (e) is consistent with your decision in (c). Be specific about what feature of the interval corresponds to “fail to reject at $\alpha = 0.05$.”

Problem 5: Conditions and Robustness (10 points)

- (a) List the conditions required for the one-sample t -test in Problem 4 to be valid.
- (b) Define what it means for a statistical procedure to be **robust**.
- (c) The one-sample t -procedure is often described as robust to departures from normality. Explain what this means in practice, and say how the sample size affects the claim.
- (d) With $n = 18$ observations in hand, describe exactly how you would use a normal Q-Q plot to check the condition in (a). What pattern would make you distrust the t -test, and what pattern would reassure you?

Problem 6: Hypothesis Testing in R (8 points)

- (a) The p-value in Problem 4(c) can be reproduced without the raw data. Explain in words what each line below computes, and report the output:

```
n <- 18; xbar <- 74.2; s <- 8.6; mu0 <- 78

tstat <- (xbar - mu0) / (s / sqrt(n))
tstat

2 * pt(-abs(tstat), df = n - 1)
```

- (b) Write one line of R using `qt()` that returns the critical value you would compare `tstat` to for a two-sided test at $\alpha = 0.05$.
- (c) If you had the 18 raw observations stored in a vector `hr`, write the single `t.test()` command that performs the test in Problem 4.