Due: ___________________________________ Name:____________________________________

Worksheet 8 – Chapter 4b

Answer the following questions logically and legibly. **Show work and give probability statements** where appropriate. Give all probabilities to 4 decimal places.

You may want to use the Normal and Exponential equations in EXCEL to answer these questions (pages 72-78 in Excel Manual).

1. Customers arrive at a local ATM on average every 4 minutes. Assume the time between arrivals follows the exponential probability distribution.
   a. What is the probability that the next customer will arrive
      i. within the next 3 minutes?

      $$P(X<3)=1-e^{-3/4}=0.5276$$

      ii. in more than 7 minutes?

      $$P(X>7)=e^{-7/4}=0.1738$$

      iii. between 4 and 8 minutes?

      $$P(4<X<8)=P(X>4)-P(X>8)=e^{-4/4}-e^{-8/4}=0.2325$$

   b. Give the Excel formula (with proper input values) for the probabilities found in part (a).
      i. =EXPON.DIST(3,0.25,TRUE)

      ii. =1-EXPON.DIST(7,0.25,TRUE)

      iii. =EXPON.DIST(8,0.25,TRUE) - EXPON.DIST(4,0.25,TRUE)

2. The commute time to work for a particular employee follows a continous uniform distribution with a minimum time of 10 minutes and maximum time of 22 minutes.
   a. Calculuate f(x).

      $$f(x)=\frac{1}{22-10}=\frac{1}{12}$$

   b. What are the mean and standard deviation for this distribution?

      $$\mu=\frac{10+22}{2}=16 \text{ minutes} \qquad \sigma=\frac{22-10}{\sqrt{12}}=3.46 \text{ minutes}$$

   c. What is the probability that the employee's next commute to work will require less than 12.5 minutes?

      $$P(X<12.5)=\frac{12.5-10}{12}=0.2083$$

d. What is the probability that the emplyee's next commute to work will require more than 14.5 minutes?

$$P(X > 14.5) = \frac{22 - 14.5}{12} = 0.6250$$

e. What is the probability that the employee's next commute to work will require between 11 and 20 minutes?

$$P(11 < X < 20) = \frac{20 - 11}{12} = 0.7500$$

f. What commute time represents the 40th percentile of this distribution?

$$P\left(x_1 \le X \le x_2\right) = \frac{x_2 - x_1}{b - a} = 0.40$$

$$\frac{x_2 - 10}{22 - 10} = \frac{x_2 - 10}{12} = 0.40$$

$$x_2 = (0.40)(12) + 10 = 14.8 \text{ minutes}$$

3. According to ESPN, the average weight of a National Football League (NFL) player in 2009 is 252.8 pounds. Assume the population standard deviation is 25 pounds. A random sample of 38 NFL players was selected.

a. Calculate the standard error of the mean.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{25}{\sqrt{38}} = \frac{25}{6.16} = 4.06$$

b. What is the probability that the same mean will be less than 246 pounds?

$$z_{246} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{246 - 252.8}{4.06} = -1.67$$

$$P\left(\bar{x} < 246\right) = P\left(z < -1.67\right) = 0.5 - 0.4525 = 0.0475$$

c. What is the probability that the sample mean will be more than 249 pounds?

$$z_{249} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{249 - 252.8}{4.06} = -0.94$$

$$P\left(\bar{x} > 249\right) = P\left(z > -0.94\right) = 0.5 + 0.3264 = 0.8264$$

d. What is the probability that the sample mean will be between 254 and 258 pounds?

$$z_{254} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{254 - 252.8}{4.06} = 0.30$$

$$z_{258} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{258 - 252.8}{4.06} = 1.28$$

$$P\left(254 \le \bar{x} \le 258\right) = P\left(0.30 \le z \le 1.28\right) = 0.3997 - 0.1179 = 0.2818$$