STAT 206: Chapter 4 (Basic Probability)

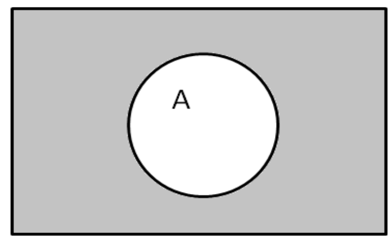
- **Ideas in Chapter 4**
  - Principles of **probability** bridge the worlds of **descriptive statistics ➔ inferential statistics**
    - What is the probability that a family **PLANS TO** purchase HDTV this year?
    - What is the probability that a family **ACTUALLY PURCHASES** HDTV this year?
    - What is the probability that a family plans to purchase **AND** actually purchases HDTV this year?
    - **GIVEN THAT** family plans to purchase, what is the probability that they actually purchase HDTV this year?
    - Does **KNOWLEDGE** that *plans* to purchase change the likelihood that family *will* purchase?
    - Etc…
- **4.1 Basic Probability Concepts**
  - **Probability** –
  - 3 types of probability:
    - *A priori*
    - Empirical probability
    - Subjective (personal)
  - Probability of an occurrence (outcome)

- Definitions:
  - **sample space**

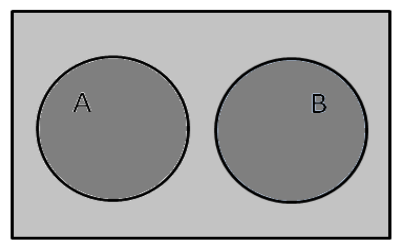
  - **Event**

  - **Joint Event –**

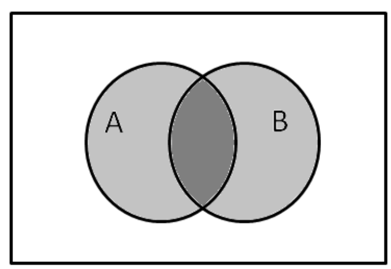
  - **Complement –**



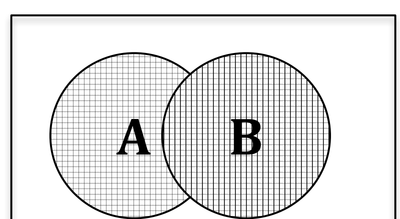
  - **Disjoint Events** –



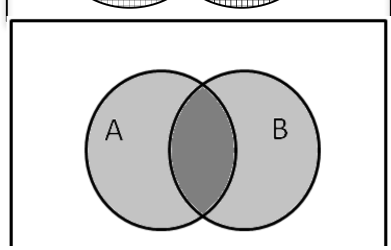
  - **Intersection of A and B –**



  - **Union of A and B** –



  - **Intersection of A and B –**



- **Joint Probability**

- **Marginal Probability –**

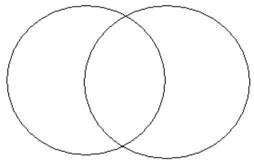
- **Example**: Probability that the family "planned to purchase" AND "actually purchased"'

	Actually Purchased		
Planned to Purchase	**Yes**	**No**	**Total**
Yes	200	50	**250**
No	100	650	**750**
Total	**300**	**700**	**1000**

- UNION Rule (General Addition Rule)
  - P(A or B) = P(A) + P(B) – P(A and B)

Example w/ Venn Diagram:

- The probability a student is in honors **math is 0.25**, in honors **science is 0.30**, and in **both is 0.20**.
- What is the probability a student is in at least one honors class?



- **4.2 Conditional Probability**

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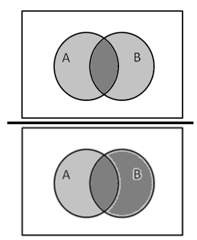
- Conditional Probability – probability of event A, given information about the occurrence of another event, B
- Probability of A given B is equal to P(A ***and*** B) divided by the P(B). That is,

$$P(A \mid B) = \frac{P(A\_and\_B)}{P(B)}$$



- Where: P(A and B) = joint probability of A ***and*** B
  P(A) = marginal probability of A
  P(B) = marginal probability of B

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- Example: What is the probability that family “Actually purchases” GIVEN THAT “Planned to purchase”?

	Actually Purchased		
Planned to Purchase	**Yes**	**No**	**Total**
Yes	200	50	**250**
No	100	650	**750**
Total	**300**	**700**	**1000**

- **EXAMPLE**: Seat belts and deaths
  - What is the probability that an individual wore a seat belt in the auto accident? That is, P(Y)?

Wore seat belt?	Survived (S)	Died (D)	Total
Yes (Y)	412,368	510	412,878
No (N)	162,527	1,601	164,128
Total	574,895	2,111	577,006

  - What is the probability that an individual survived in the auto accident? That is, P(S)?

  - What is the probability that an individual did NOT survive the auto accident? That is, P(D)?

  - What is the probability that an individual wore a seat belt **and** survived in the auto accident? That is, P(S and Y)?

  - What is the probability that an individual wore a seat belt **or** survived in the auto accident? That is, P(S or Y)?

  - What is the probability of surviving GIVEN that the person wore a seatbelt? That is, P(S |Y)?

- **Decision Trees** –alternative to Contingency Tables

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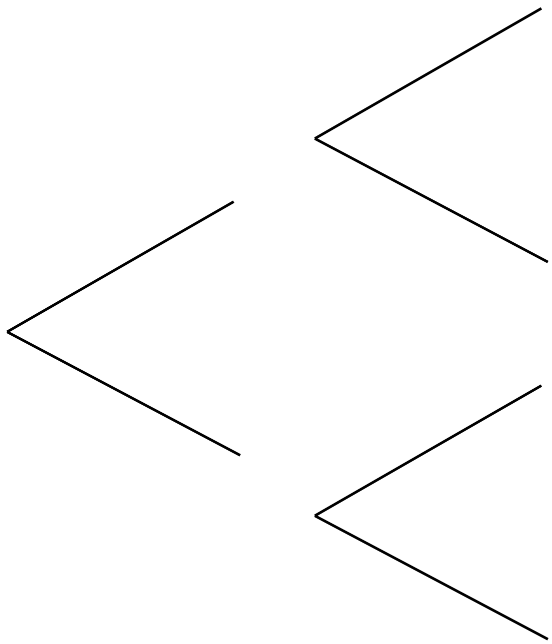

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- **Independence of events**
  - Two events, A and B, are independent if
  - Two events, A and B, are independent if and only if
  - If one of the following is true, the all three are true:
    1.
    2.
    3.

- **Example**: Of the 300 households that purchased HDTV, they either purchased a standard refresh rate or a faster refresh rate. The contingency table shows satisfaction.
  - What is the probability that a family was satisfied with the purchase?

	Satisfied with Purchase		
TV Refresh Rate	**Yes**	**No**	**Total**
Faster	64	16	**80**
Standard	176	44	**220**
Total	**240**	**60**	**300**

  - What is the probability that a family that bought a TV with faster refresh rate was satisfied with the purchase?

  - Are the events “being satisfied with the purchase” and the “refresh rate of the TV” independent?

- **General Multiplication Rule**

$$P(A | B) = \frac{P(A\_and\_B)}{P(B)}$$

  - **Example**: Previous contingency table, but only considering FASTER rate. Suppose 2 households are chosen at random from the 80 households. Find the probability that both households are satisfied with their purchases. Let: A = 2nd household is satisfied **AND** B = 1st household is satisfied

	Satisfied with Purchase		
TV Refresh Rate	**Yes**	**No**	**Total**
Faster	64	16	**80**
Standard	176	44	**220**
Total	**240**	**60**	**300**

- **4.4 Counting Rules**
  - **Rule 1**: If any of k different mutually exclusive and collectively exhaustive evens can occur on each of n trials, the number of possible outcomes is equal to k^n
  - **Rule 2**: If there are k_1 events on the first trial, $k2$ events on the second trial, … and k_n events on the nth trial, then the number of possible outcomes is $(k_1)(k_2)\ldots(k_n)$
  - **Rule 3**: The number of ways that all n items can be arranged in order is $n! = (n)(n-1)(n-2)\ldots(1)$
  - **Rule 4** (PERMUTATIONS): The number of ways arranging x objects selected from n objects **in order** is equal to ${}_nP_x = \frac{n!}{(n-x)!}$ (where n = total number of objects, x = number of objects to be arranged, n! = n factorial = n(n-1)(n-2)…(1), P = symbol for permutations)
  - **Rule 5** (COMBINATIONS): The number of ways of selecting x objects from n objects**, irrespective of order** is equal to: ${}_nC_x = \frac{n!}{x!(n-x)!}$ (where where n = total number of objects, x = number of objects to be arranged, n! = n factorial = n(n-1)(n-2)…(1), C = symbol for combinations)

- **4.5 Ethical Issues and Probability**