Ideas in Chapter 5:
- Concept and characteristics of probability distributions
  - Binomial distribution
  - Poisson distribution
  - Maybe covariance
  - Maybe Hypergeometric
- Important to remember:
  - What is a discrete numeric variable?

- Counting rules

5.1 Probability Distribution for a Discrete Variable

- **Discrete variables:** Have numerical values that arise from a counting process
- **Probability distribution for a discrete random variable:**
  Mutually exclusive list of all the possible numerical outcomes along with the probability of each outcome

- Example:
  - Number of interruptions per day in a large computer network

<table>
<thead>
<tr>
<th>Interruptions in a day</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.35</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
</tr>
</tbody>
</table>
• Expected value, $\mu$, of a Discrete Variable
  • Expected value, $\mu$, of a discrete variable, where $x_i = i^{th}$ value of the discrete variable $X$ and $P(X=x_i)$ = probability of the occurrence of the $i^{th}$ value of $X$ is:

\[
E(x) = \sum xP(x)
\]

<table>
<thead>
<tr>
<th>Interruptions in a day</th>
<th>Probability</th>
<th>$(x_i)(P(X=x_i))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

• Variance of a Discrete Variable
  • Variance, $\sigma^2$, of a discrete variable, where $x_i = i^{th}$ value of the discrete variable $X$ and $P(X=x_i)$ = probability of the occurrence the $i^{th}$ value of $X$ is:

\[
\sigma^2 = \sum_{i=1}^{N} [x_i - E(X)]^2 P(X = x_i)
\]

<table>
<thead>
<tr>
<th>Interruptions in a day</th>
<th>Probability</th>
<th>$x_i P(X=x_i)$</th>
<th>$[x_i - E(x)]^2$</th>
<th>$[x_i - E(x)]^2 P(X=x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.35</td>
<td>0.00</td>
<td>[]</td>
<td>[]</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Standard Deviation of a Discrete Variable
  • Standard deviation, $\sigma$, of a discrete variable, where $x_i = i^{th}$ value of the discrete variable $X$ and $P(X=x_i)$ = probability of the occurrence of the $i^{th}$ value of $X$ is:

\[
\sigma = \sqrt{\sigma^2}
\]
• **5.3 Binomial Distribution**
  
  • Properties of a Binomial
  
  • A fixed number of observations, n

  • Each observation is categorized as to whether or not the “event of interest” occurred

  • Constant probability for the event of interest occurring (π) for each observation

• **Examples of business applications:**

• **Counting Techniques for Binomial**
  
  • Suppose the event of interest is obtaining heads on the toss of a fair coin. You are to toss the coin three times. In how many ways can you get two heads?

  • The number of combinations of selecting x objects out of n objects is

• **Example:** How many possible 3 scoop combinations could you create at an ice cream parlor if you have 31 flavors to select from and no flavor can be used more than once in the 3 scoops?
  
  • The total choices is n = 31, and we select X = 3
• Binomial Distribution

- \( P(X=x \mid n, \pi) = \) probability of \( x \) events of interest (i.e., \( x \) “successes”) in \( n \) trials, with the probability of a “success” of \( \pi \) for each trial
- \( x = \) number of “events of interest” (“successes”) in sample (\( x = 0, 1, 2, \ldots, n \))
- \( n = \) sample size (number of trials or observations)
- \( \pi = \) probability of “event of interest” (“success”)

• Example: Flip a coin four times, let \( X=\# \) heads:

\[
x = \\
n = \\
\pi = \\
\]

• EXAMPLE: Suppose the probability of purchasing a defective computer is 0.02. What is the probability of purchasing 2 defective computers in a group of 10?

• What do we know?

- \( P(X=x \mid n, \pi) = \)
• Shape of the Binomial Distribution
  • Controlled by the values of $\pi$ and $n$

**Shape of the Binomial Distribution**

• Controlled by the values of $\pi$ and $n$
  • For example, $n = 5$ and $\pi = 0.10$
  • Another example, $n = 5$ and $\pi = 0.50$

**Binomial Tables**

[Table of Binomial Distribution values]


• **Binomial Distribution Characteristics**
  • Mean: $\mu = E(X) = n\pi$
  • Variance: $\sigma^2 = n\pi(1-\pi)$
  • Standard deviation: $\sigma = \sqrt{n\pi(1-\pi)}$

  • Where: $n =$ sample size (# trials)
    $\pi =$ probability of the event of interest (success) for any trial
    $(1-\pi) =$ probability of no success for any trial
Excel Can Be Used To Calculate The Binomial Distribution

Excel Can Be Used To Calculate The Binomial Distribution

PEARSON slide (Chapter 5, #35, mostly…)

=BINOM.DIST(x,4,0.30, cumulative?)

"cumulative?" = FALSE

"cumulative?" = TRUE