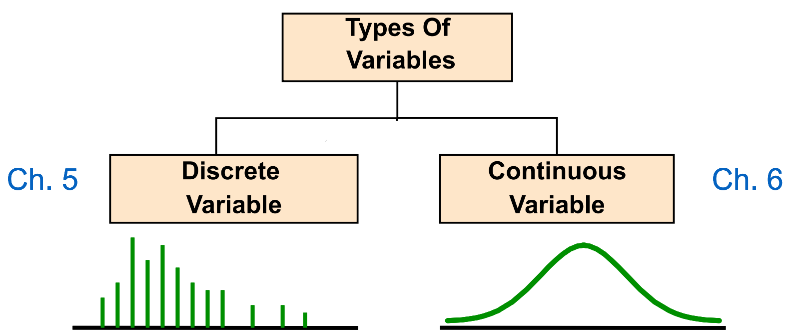
Ideas in Chapter 5:

- Concept and characteristics of probability distributions
  - Binomial distribution
  - Poisson distribution
  - Maybe covariance
  - Maybe Hypergeometric
- Important to remember:
  - What is a discrete numeric variable?

  - Counting rules

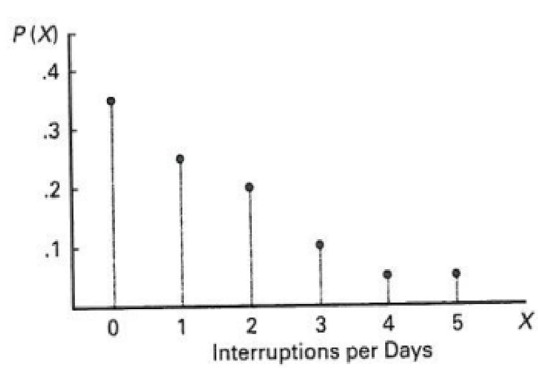
5.1 Probability Distribution for a Discrete Variable

- **Discrete variables:**
  Have **numerical values** that arise from a **counting** process
- **Probability distribution for a discrete random variable:**
  Mutually exclusive list of all the possible numerical outcomes along with the probability of each outcome



- Example:
- Number of interruptions per day in a large computer network

Interruptions in a day	Probability
0	0.35
1	0.25
2	0.20
3	0.10
4	0.05
5	0.05



Counting Process?

Mutually Exclusive?

Valid Probabilities?

Sum to one (1)?

- Expected value, μ, of a Discrete Variable
  - Expected value, μ, of a discrete variable, where x_i = i^{th} value of the discrete variable X and P(X=x_i) = probability of the occurrence of the i^{th} value of X is:

$$E(x) = \sum xP(x)$$

Interruptions in a day	Probability	$(x_i)(P(X=x_i))$	
0	0.35		
1	0.25		
2	0.20		
3	0.10		
4	0.05		
5	0.05		
	1.00		

- Variance of a Discrete Variable
  - Variance, σ^2, of a discrete variable, where x_i = i^{th} value of the discrete variable X and P(X=x_i) = probability of the occurrence the i^{th} value of X is:

$$\boldsymbol{\sigma^2 = \sum_{i=1}^{N} [x_i - E(X)]^2 P(X = x_i)}$$

Interruptions in a day	**Probability**	**x_i P(X=x_i)**	**$[x_i\text{-}E(x)]^2$**	**$[x_i\text{-}E(x)]^2P(X=x_i)$**	
0	**0.35**	**0.00**			
1	**0.25**	**0.25**			
2	**0.20**	**0.40**			
3	**0.10**	**0.30**			
4	**0.05**	**0.20**			
5	**0.05**	**0.25**			
	1.00	**1.40**			

- Standard Deviation of a Discrete Variable
  - Standard deviation, σ, of a discrete variable, where x_i = i^{th} value of the discrete variable X and P(X=x_i) = probability of the occurrence of the i^{th} value of X is:

$$\boldsymbol{\sigma = \sqrt{\sigma^2}}$$

- **5.3 Binomial Distribution**
  - Properties of a Binomial
    - A fixed number of observations, n
    - Each observation is categorized as to whether or not the “event of interest” occurred
    - Constant probability for the event of interest occurring (π) for each observation
  - Examples of business applications:
  - **Counting Techniques for Binomial**
    - Suppose the event of interest is obtaining heads on the toss of a fair coin. You are to toss the coin three times. In how many ways can you get two heads?
    - The number of combinations of selecting x objects out of n objects is
  - **Example:** How many possible 3 scoop combinations could you create at an ice cream parlor if you have 31 flavors to select from and no flavor can be used more than once in the 3 scoops?
    - The total choices is n = 31, and we select X = 3

- **Binomial Distribution**

  - $P(X=x|n,\pi)$ = probability of x events of interest (i.e., x "successes") in n trials, with the probability of a "success" of π for each trial
  - x = number of "events of interest" ("successes") in sample (x = 0, 1, 2, …, n)
  - n = sample size (number of trials or observations)
  - π = probability of "event of interest" ("success")

- Example: Flip a coin four times, let X=# heads:

  x =

  n =

  π =

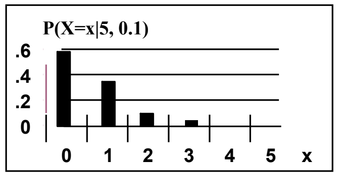
- EXAMPLE: Suppose the probability of purchasing a defective computer is 0.02. What is the probability of purchasing 2 defective computers in a group of 10?
  - What do we know?

  - $P(X=x|n,\pi)=$

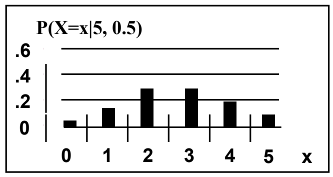
- Shape of the Binomial Distribution
  - Controlled by the values of π and n

Shape of the Binomial Distribution

- Controlled by the values of π and **n**
- For example,
  n = 5 and π = 0.10



- Another example,
  n = 5 and π = 0.50



15

Binomial Tables

n	r	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160	.123	.090	.063
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480	.455	.420	.375
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360	.423	.490	.563
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064	.043	.027	.016
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288	.239	.189	.141
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432	.444	.441	.422
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216	.275	.343	.422
4	0	.961	.815	.656	.522	.410	.316	.240	.179	.130	.092	.062	.041	.026	.015	.008	.004
	1	.039	.171	.292	.368	.410	.422	.412	.384	.346	.300	.250	.200	.154	.112	.076	.047
	2	.001	.014	.049	.098	.154	.211	.265	.311	.346	.368	.375	.368	.346	.311	.265	.211
	3	.000	.000	.004	.011	.026	.047	.076	.112	.154	.200	.250	.300	.346	.384	.412	.422
	4	.000	.000	.000	.001	.002	.004	.008	.015	.026	.041	.062	.092	.130	.179	.240	.316
5	0	.951	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031	.019	.010	.005	.002	.001
	1	.048	.204	.328	.392	.410	.396	.360	.312	.259	.206	.156	.113	.077	.049	.028	.015
	2	.001	.021	.073	.138	.205	.264	.309	.336	.346	.337	.312	.276	.230	.181	.132	.088
	3	.000	.001	.008	.024	.051	.088	.132	.181	.230	.276	.312	.337	.346	.336	.309	.264
	4	.000	.000	.000	.002	.006	.015	.028	.049	.077	.113	.156	.206	.259	.312	.360	.396
	5	.000	.000	.000	.000	.000	.001	.002	.005	.010	.019	.031	.050	.078	.116	.168	.237
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004	.002	.001	.000
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037	.020	.010	.004
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138	.095	.060	.033
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276	.236	.185	.132
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311	.328	.324	.297
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187	.244	.303	.356
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047	.075	.118	.178
7	0	.932	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008	.004	.002	.001	.000	.000
	1	.066	.257	.372	.396	.367	.311	.247	.185	.131	.087	.055	.032	.017	.008	.004	.001
	2	.002	.041	.124	.210	.275	.311	.318	.299	.261	.214	.164	.117	.077	.047	.025	.012
	3	.000	.004	.023	.062	.115	.173	.227	.268	.290	.292	.273	.239	.194	.144	.097	.058
	4	.000	.000	.003	.011	.029	.058	.097	.144	.194	.239	.273	.292	.290	;268	.227	.173
	5	.000	.000	.000	.001	.004	.012	.025	.047	.077	.117	.164	.214	.261	.299	.318	.311
	6	.000	.000	.000	.000	.000	.001	.004	.008	.017	.032	.055	.087	.131	.185	.247	.311
	7	.000	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.015	.028	.049	.082	.133

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- **Binomial Distribution Characteristics**
  - Mean: $\mu = E(X) = n\pi$
  - Variance: $\sigma^2 = n\pi(1-\pi)$
  - Standard deviation: $\sigma = \sqrt{n\pi(1-\pi)}$
  - Where: n = sample size (# trials)
    π=probability of the event of interest (success) for any trial
    (1- π)=probability of no success for any trial

- Excel Can Be Used To Calculate The Binomial Distribution

Excel Can Be Used To Calculate The Binomial Distribution

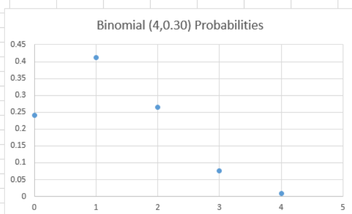
	A	B	
1	**Binomial Probabilities**		
2			
3	**Data**		
4	Sample size	4	
5	Probability of an event of interest	0.1	
6			
7	**Statistics**		
8	Mean	0.4	=B4 * B5
9	Variance	0.36	=B8 * (1 - B5)
10	Standard deviation	0.6	=SQRT(B9)
11			
12	**Binomial Probabilities Table**		
13	***X***	***P(X)***	
14	0	0.6561	=BINOM.DIST(A14, B4, B5, FALSE)
15	1	0.2916	=BINOM.DIST(A15, B4, B5, FALSE)
16	2	0.0486	=BINOM.DIST(A16, B4, B5, FALSE)
17	3	0.0036	=BINOM.DIST(A17, B4, B5, FALSE)
18	4	0.0001	=BINOM.DIST(A18, B4, B5, FALSE)

=BINOM.DIST(<#successes>,<#trials>,<probability_success>,<cumulative?>)

Pearson slide (Chapter 5, #35, mostly…)

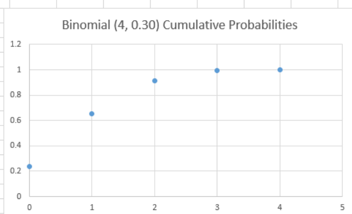
=BINOM.DIST(*x*,4,0.30,***cumulative?***)

n=	4
p(success)=	0.3
# successes:	P(x)
0	0.2401
1	0.4116
2	0.2646
3	0.0756
4	0.0081
	1.0000



"***cumulative?***" = FALSE

n=	4
p(success)=	0.3
# successes:	
0	0.2401
1	0.6517
2	0.9163
3	0.9919
4	1



"***cumulative?***" = TRUE

19