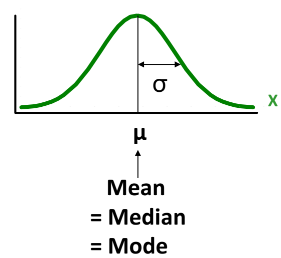
Ideas in Chapter 6:

- Normal distribution
  - Compute probabilities from the normal distribution
  - How to use the normal distribution to solve business problems
  - (Maybe) How to use the normal probability plot to determine whether a set of data is approximately normally distributed
- Brief look at and discussion of:
  - Computing probabilities from the uniform distribution
  - Computing probabilities from the exponential distribution
- Important to remember:
  - What is a continuous numeric variable?

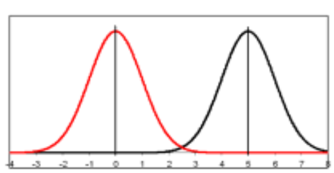
- **6.2 Normal Distribution**
  - Bell Shaped
  - Symmetrical
  - Mean, Median and Mode are Equal
  - Location is determined by the standard deviation, σ
  - The random variable has an infinite theoretical range - ∞ to + ∞



Questions:

Pictured at the right are two different normal distributions. Which is different between the two distributions?

A. Mean
B. Standard deviation
C. Both



Which is different between the two normal distributions to the right?

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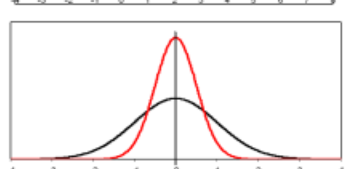

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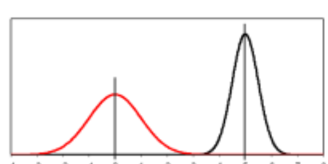
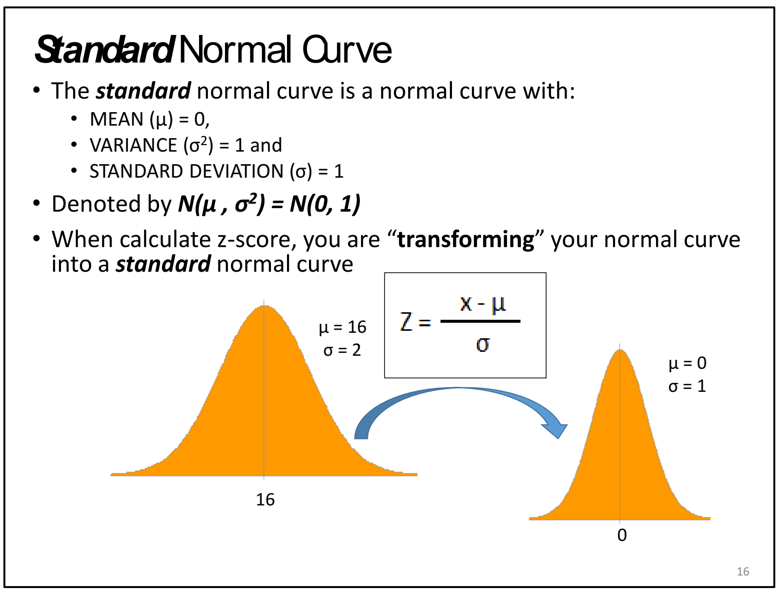


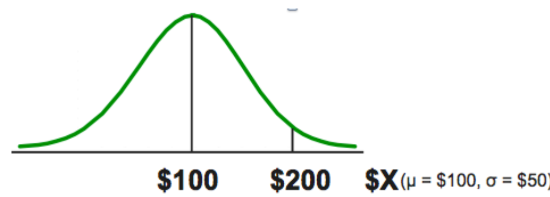
- **Normal Probability Density Function**
  - Formula for the normal probability density function is: $f(X) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{(X-\mu)}{\sigma}\right)^2}$
  - Where:
    - e = the mathematical constant approximated by 2.71828
    - π = the mathematical constant approximated by 3.14159
    - μ = the population mean
    - σ = the population standard deviation
    - X = any value of the continuous variable Standardized Normal

- **Standardized Normal**
  - Any normal distribution (with any mean and standard deviation combination) can be transformed into the standardized normal distribution (Z)
  - To compute normal probabilities need to transform X units into Z units
  - $z = \frac{(x-\mu)}{\sigma} \approx \frac{x-\bar{X}}{s}$
  - The standardized normal distribution (Z) has a mean of 0 and a standard deviation of 1

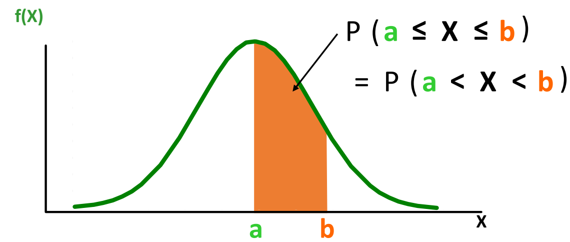


- **Translation to the Standardized Normal**
  - To translate from X to the standardized normal "Z" distribution, subtract the mean, μ, and divide by the standard deviation, σ. That is, $z = \frac{(x-\mu)}{\sigma} \approx \frac{x-\bar{X}}{s}$
  - The standard Normal Z distribution always has mean = μ = 0 and standard deviation = σ = 1
    - Values above the mean have positive Z-values
    - Values below the mean have negative Z-values
  - Notice we are using the ***POPULATION PARAMETER*** values μ and σ

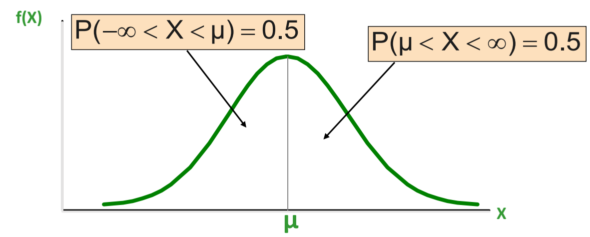
- **EXAMPLE**: If X is distributed normally with mean of $100 and standard deviation of $50 what is the Z value for x = $200?



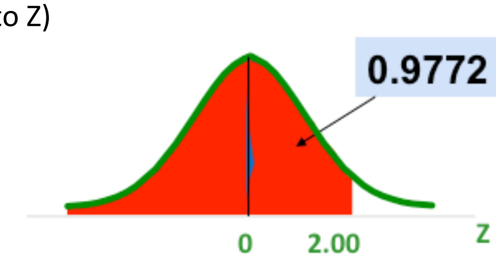
- **Finding Normal Probabilities**
  - Probability is measured by ***AREA UNDER THE CURVE***
  - **NOTE**: PROBABILITY of an individual point is zero

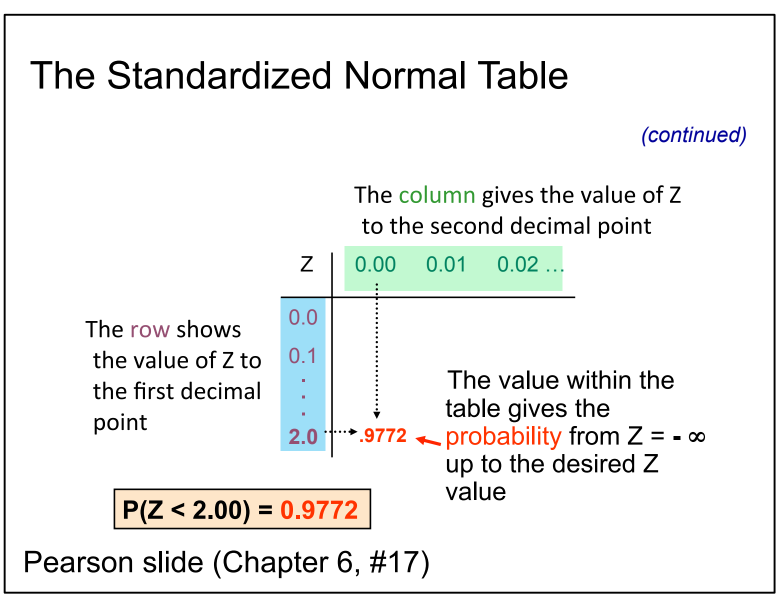


- **Probability under the curve**
  - TOTAL area under any density curve = 1
  - Normal distribution is symmetric so…



- **Standardized Normal Table**
  - Cumulative Standardized Normal table in the textbook (***Appendix table E.2***) gives the probability less than a desired value of Z (i.e., from negative infinity to Z)
  - **EXAMPLE**: P(Z < 2.00)





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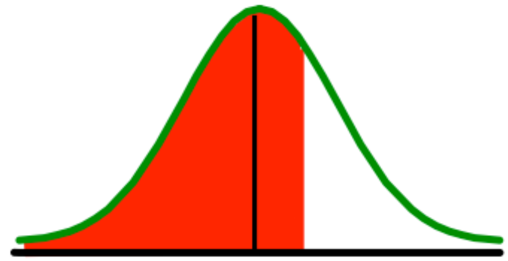
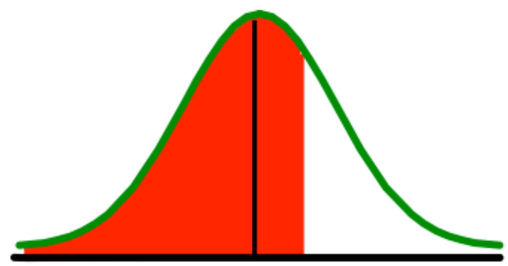
- **(General) Procedures for Finding Normal Probabilities**
  - To find P(a < X < b) when X is distributed normally:
    - Draw the normal curve for the problem in terms of X
    - Translate X-values to Z-values
    - Use the Standardized Normal Table
  - **EXAMPLE**: Let X represent the time it takes (in seconds) to download an image file from the internet. Suppose X is normal with a mean, μ = 18.0 seconds and a standard deviation, σ = 5.0 seconds. Find the probability that it takes less than 18.6 seconds to download an image file. That is, find P(X < 18.6).



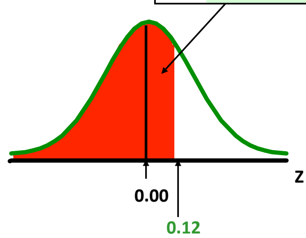


Solution: Finding P(Z < 0.12)

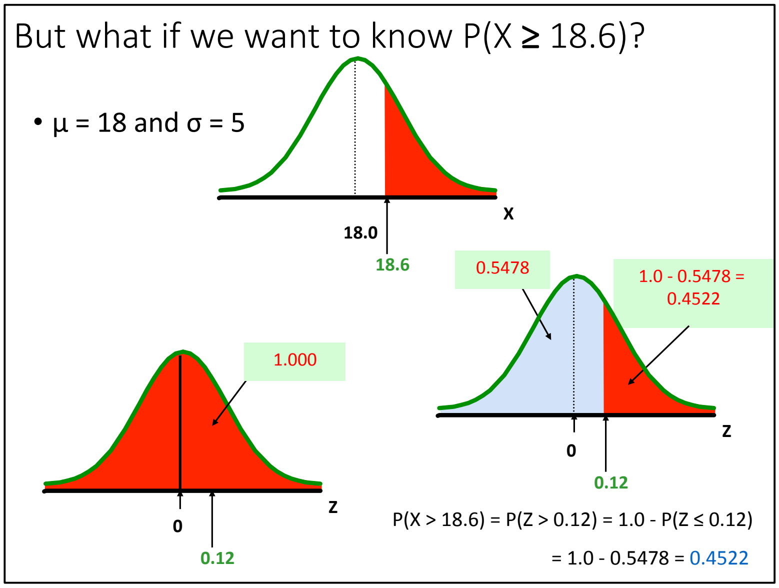
Standardized Normal Probability Table (Portion)

Z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

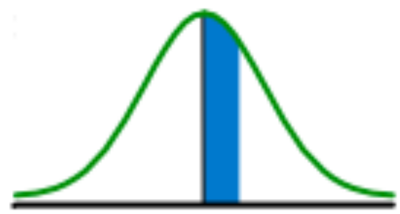
P(X < 18.6)
= P(Z < 0.12)
0.5478



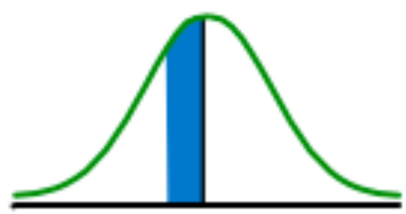
Pearson slide (Chapter 6, #21)



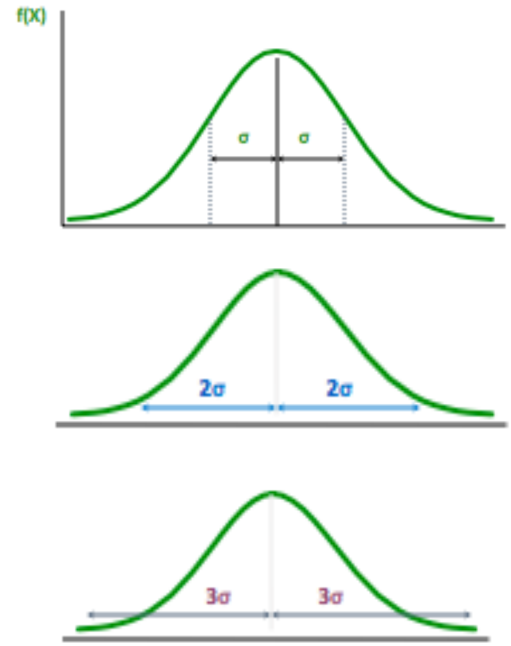
- **Finding a Normal Probability Between Two Values**
  - Suppose X is normal with mean 18.0 and standard deviation 5.0. Find P(18 < X < 18.6)



- Probabilities in the Lower Tail
  - Suppose X is normal with μ = 18 and σ = 5



- **Empirical Rule**
- What can we say about the distribution of values around the mean? For any normal distribution:



- **What if you are given a Normal Probability, and you need to find x?**

  - Steps to find the X value for a known probability:

EXAMPLE: Let X represent the time it takes (in seconds) to download an image file from the internet. Suppose X is normal with mean 18.0 and standard deviation 5.0. Find X such that 20% of download times are less than X.

Excel Can Be Used To Find Normal Probabilities

- Find P(X < 9) where X is normal with a mean of 7 and standard deviation of 2

	A	B	
1	**Normal Probabilities**		
2			
3	**Common Data**		
4	Mean	7	
5	Standard Deviation	2	
6			
7	**Probability for X <=**		
8	X Value	7	
9	Z Value	0	**=STANDARDIZE(B8, B4, B5)**
10	P(X<=7)	0.5000	**=NORM.DIST(B8, B4, B5, TRUE)**
11			
12	**Probability for X >**		
13	X Value	9	
14	Z Value	1	**=STANDARDIZE(B13, B4, B5)**
15	P(X>9)	0.1587	**=1 - NORM.DIST(B13, B4, B5, TRUE)**

6.3 Evaluating Normality

- Not all continuous distributions are normal

  ➔ important to evaluate how well data are approximated by a normal distribution
- Normally distributed data should approximate the theoretical normal distribution:
  - Bell-shaped (symmetrical) where the mean is equal to the median
  - Empirical rule applies
  - Interquartile range ~ 1.33 standard deviations

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Comparing data characteristics to theoretical properties

- Construct charts or graphs
  - For small- or moderate-sized data sets, construct stem-and-leaf or boxplot to check for symmetry
  - For large data sets, does the histogram or polygon appear bell-shaped?
- Compute descriptive summary measures
  - Do the mean, median and mode have similar values?
  - Is the interquartile range approximately 1.33σ?
  - Is the range approximately 6σ?
- Observe the distribution of the data set
  - Do approximately 2/3 of the observations lie within mean ±1 standard deviation?
  - Do approximately 80% of the observations lie within mean ±1.28 standard deviations?
  - Do approximately 95% of the observations lie within mean ±2 standard deviations?

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Comparing data characteristics to theoretical properties

- Evaluate normal probability plot
  - Is the normal probability plot approximately linear (i.e. a straight line) with positive slope?

So what is this??????

- Constructing a Normal Probability Plot
  - Arrange data into ordered array
  - Find corresponding standardized normal quantile values (Z)
  - Plot the pairs of points with observed data values (X) on the vertical axis and the standardized normal quantile values (Z) on the horizontal axis
  - Evaluate the plot for evidence of linearity

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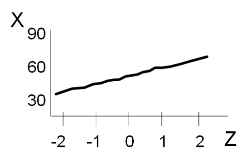
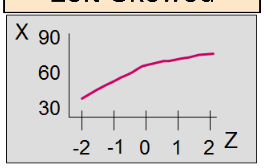
The Normal Probability Plot Interpretation

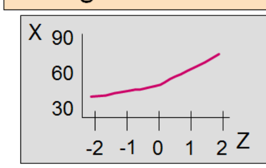
A normal probability plot for data from a normal distribution will be approximately linear:



Left-Skewed



Right-Skewed



- **6.4 Uniform Distribution**
  - The uniform distribution is a probability distribution that has equal probabilities for all possible outcomes of the random variable (also called a rectangular distribution)

  - **Uniform** Probability Density Function and Properties:

$$f(X) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le X \le b \\ 0 & \text{otherwise} \end{cases}$$

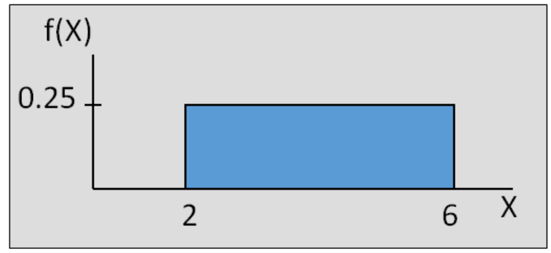
Mean of the Uniform Distribution: $\mu = \dfrac{a+b}{2}$

Standard deviation: $\sigma = \sqrt{\dfrac{(b-a)^2}{12}}$

where

f(X) = value of the density function at any X value
a = minimum value of X
b = maximum value of X

- **Example**: Uniform probability distribution over the range $2 \le X \le 6$:

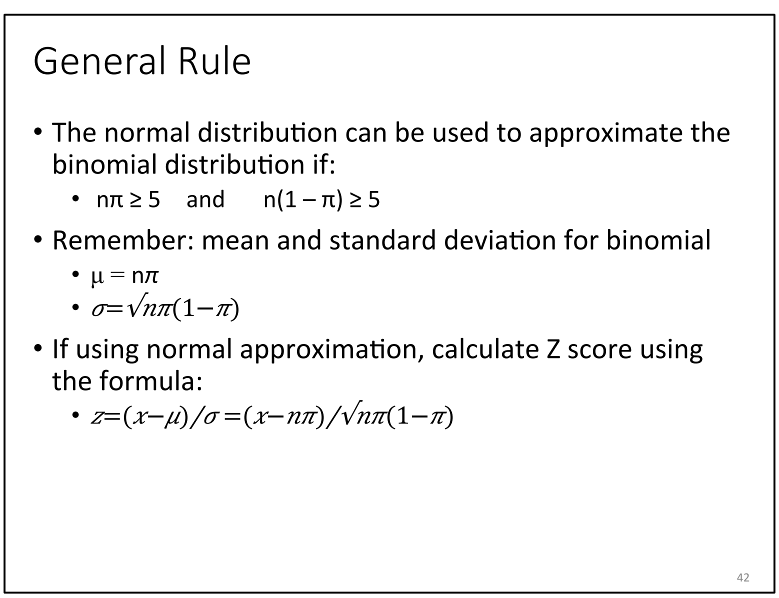
  

  μ =

  σ =

  $P(3 \le X \le 5)$ =

- **6.6 Normal Approximation to Binomial**
  - The binomial distribution is a discrete distribution, but the normal is continuous
  - To use the normal to approximate the binomial, accuracy is improved if you use a correction for continuity adjustment
  - **EXAMPLE**: X is discrete in a binomial distribution, so $P(X = 4)$ can be approximated with a continuous normal distribution by finding
  - The closer π is to 0.5, the better the normal approximation to the binomial
  - The larger the sample size n, the better the normal approximation to the binomial



- **EXAMPLE:** If n = 1000 and $\pi = 0.2$, what is $P(X \leq 180)$?
-