STAT 201 Chapter 9.1-9.2

Hypothesis Testing for Proportion

Confidence Intervals to Testing

- As we see in chapter 8.1 and 8.2 we can come up with interesting observations given our confidence intervals

- Next we will learn how to formally test whether or not the population proportion is a particular value based on our sample proportion

Hypothesis

- A **Hypothesis** is a proposition assumed as a premise in an argument. It's a statement regarding a characteristic of one or more populations.

- **Hypothesis testing** is a procedure based on evidence found in a sample to test hypothesis

Hypothesis

- The **null hypothesis, $\boldsymbol{H_0}$,** is a statement to be tested. The null hypothesis is a statement of no change, no effect or no difference and is assumed true until evidence indicates otherwise

- The **alternative hypothesis, $\boldsymbol{H_1}$ or $\boldsymbol{H_a}$,** is a statement that we are trying to find evidence to support

Hypothesis

1. *Two-tailed test*
   - $H_0: parameter = some\ value$
   - $H_a: parameter \neq some\ value$
2. *Left-tailed test*
   - $H_0: parameter \geq some\ value$
   - $H_a: parameter < some\ value$
3. *Right-tailed test*
   - $H_0: parameter \leq some\ value$
   - $H_a: parameter > some\ value$

Hypothesis Test for Proportions: Step 1

- We are interested in testing whether the population proportion, p, is equal, or great, or less than p_o.
- Step 1 is to know what hypothesis you wan to test.

Two-tailed test	*Left-tailed test*	*Right-tailed test*
$H_o: p = p_o$	$H_o: p \geq p_o$	$H_o: p \leq p_o$
$H_a: p \neq p_o$	$H_a: p < p_o$	$H_a: p > p_o$

Hypothesis Test for Proportions: Step 2

- Check the assumptions:
  1. The variable must be categorical
  2. The data should be obtained using randomization
  3. The sample size is sufficiently large where p_o is the testing value satisfying
     - $np_o \geq 15$
     - $n(1 - p_o) \geq 15$
     - It is safe to assume the distribution of p_o has a bell shaped distribution

Hypothesis Test for Proportions: Step 3

- Calculate Test Statistic, z*
  - The test statistic measures how different the sample proportion we have is from the null hypothesis
  - We calculate the z-score by assuming that p_o is the population proportion

$$z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o(1 - p_o)}{n}}}$$

Hypothesis Test for Proportions: Step 4

- Determine the P-value
  - What is P-value?
  - The P-value describes how unusual/unlikely the sample data would be if H_o were true.
  - z* is the test statistic from step 3

Alternative Hypothesis	Probability	Formula for the P-value
$H_a: p > p_o$	Right tail	P(Z>z*)
$H_a: p < p_o$	Left tail	P(Z<z*)
$H_a: p \neq p_o$	Two-tail	2*P(Z<-|z*|)

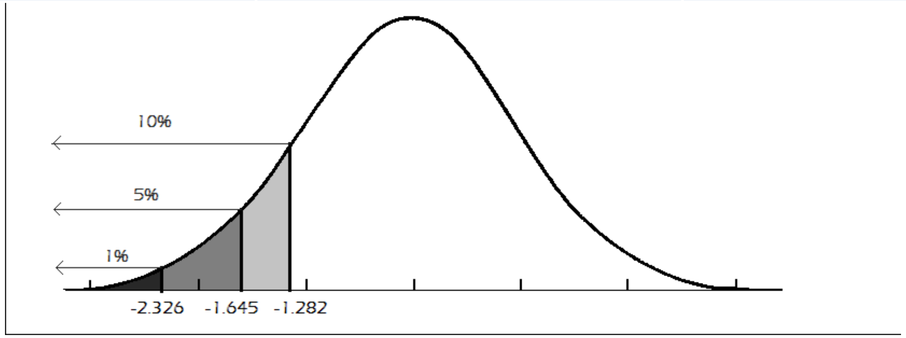
Hypothesis Test for Proportions: Step 5

- Summarize the test by reporting and interpreting the P-value
  - Smaller p-values give stronger evidence against H_o
- If p-value$\leq (1 - confidence) = \alpha$
  - Reject H_o, with a p-value = ____, we have sufficient evidence that the alternative hypothesis might be true.
- If p-value$> (1 - confidence) = \alpha$
  - Fail to reject H_o, with a p-value = ____, we do not have sufficient evidence that the alternative hypothesis might be true.

Hypothesis Test for Proportions: Step 5 (cont.)

- For a left tailed test: $H_a: p < p_0$ → We have rejection regions for H_o are as follows
  - Note: all of the rejection region is in the left tail, where $\hat{p}$ is much smaller than p_0

Confidence	Reject (test stat)	Reject (p-value)
0.90	Test-stat<-1.282	P-value<.1
0.95	Test-stat<-1.645	P-value<.05
0.99	Test-stat<-2.326	P-value<.01



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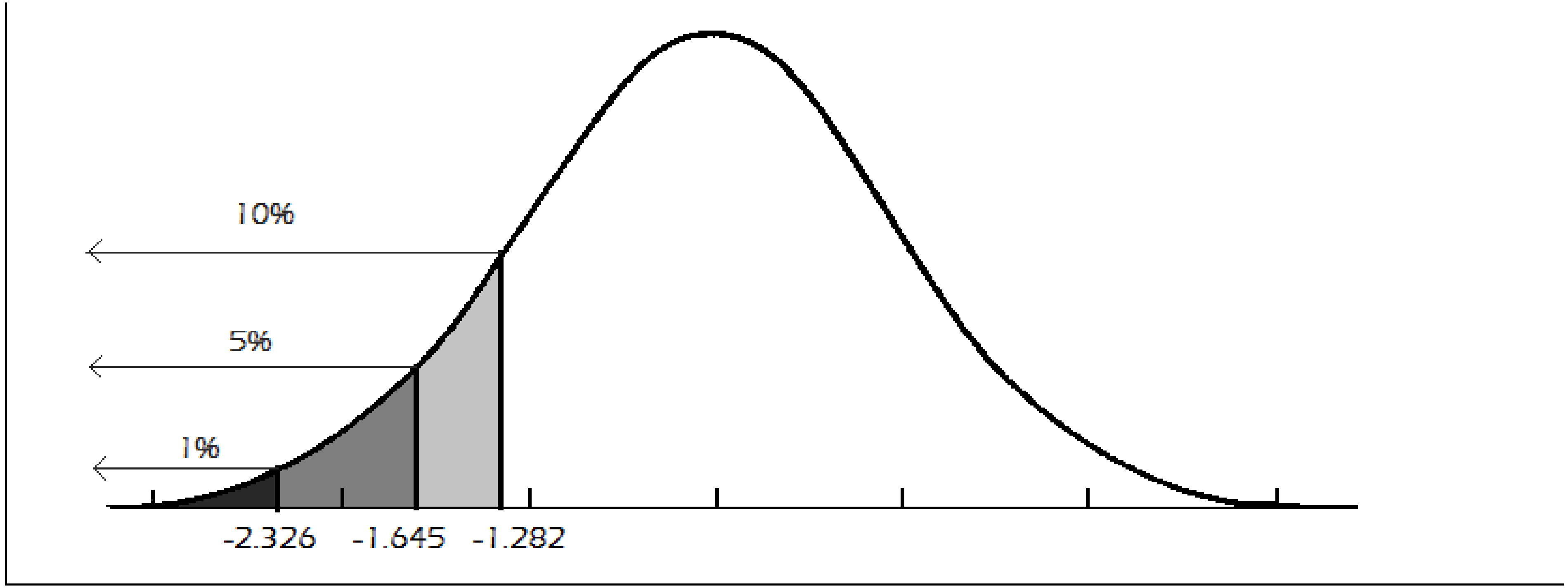

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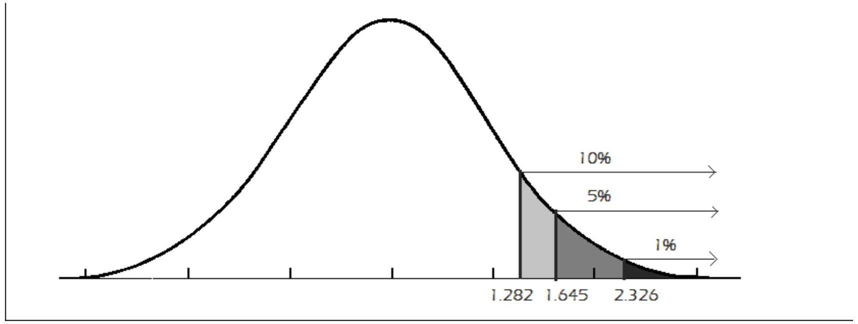
0.95	Test-stat<-1.645	P-value<.05
0.99	Test-stat<-2.326	P-value<.01

Hypothesis Test for Proportions: Step 5 (cont.)

- For a right tailed test: $H_a: p > p_0$ → We have rejection regions for H_o are as follows
  - Note: all of the rejection region is in the right tail, where $\hat{p}$ is much larger than p_0

Confidence	Reject (test stat)	Reject (p-value)
0.90	Test-stat>1.282	P-value<.1
0.95	Test-stat>1.645	P-value<.05
0.99	Test-stat>2.326	P-value<.01



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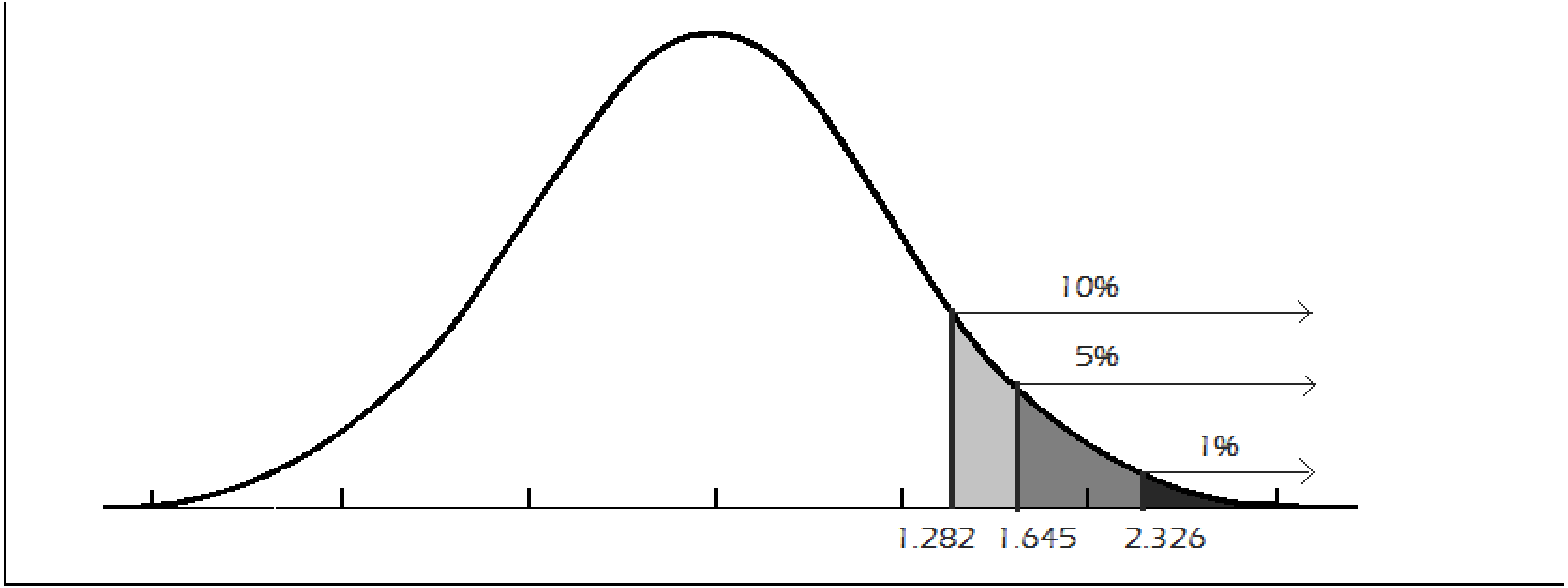
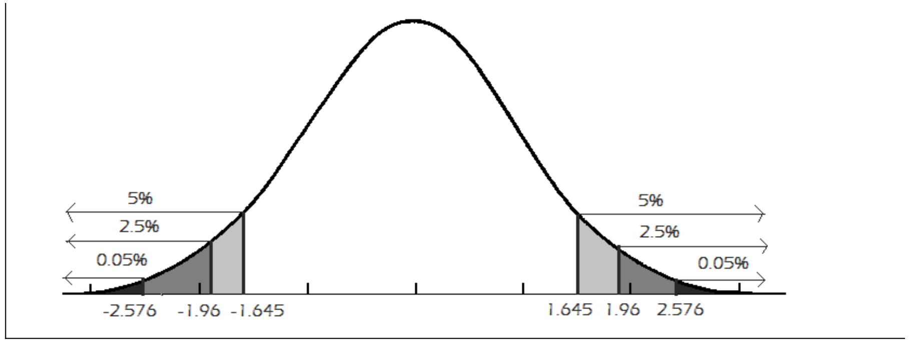


Confidence	Reject (test stat)	Reject (p-value)
0.90	Test-stat>1.282	P-value<.1
0.95	Test-stat>1.645	P-value<.05
0.99	Test-stat>2.326	P-value<.01

Hypothesis Test for Proportions: Step 5 (cont.)

- For a two tailed test: $H_a: p \neq p_0$ → We have rejection regions for H_o are as follows
  - Note: here we split the rejection region into both tails, where $\hat{p}$ is very different from p_0

Confidence	Reject (test stat)	Reject (p-value)
0.90	|Test-stat|<1.645	P-value<.1
0.95	|Test-stat|<1.960	P-value<.05
0.99	|Test-stat|<2.576	P-value<.01



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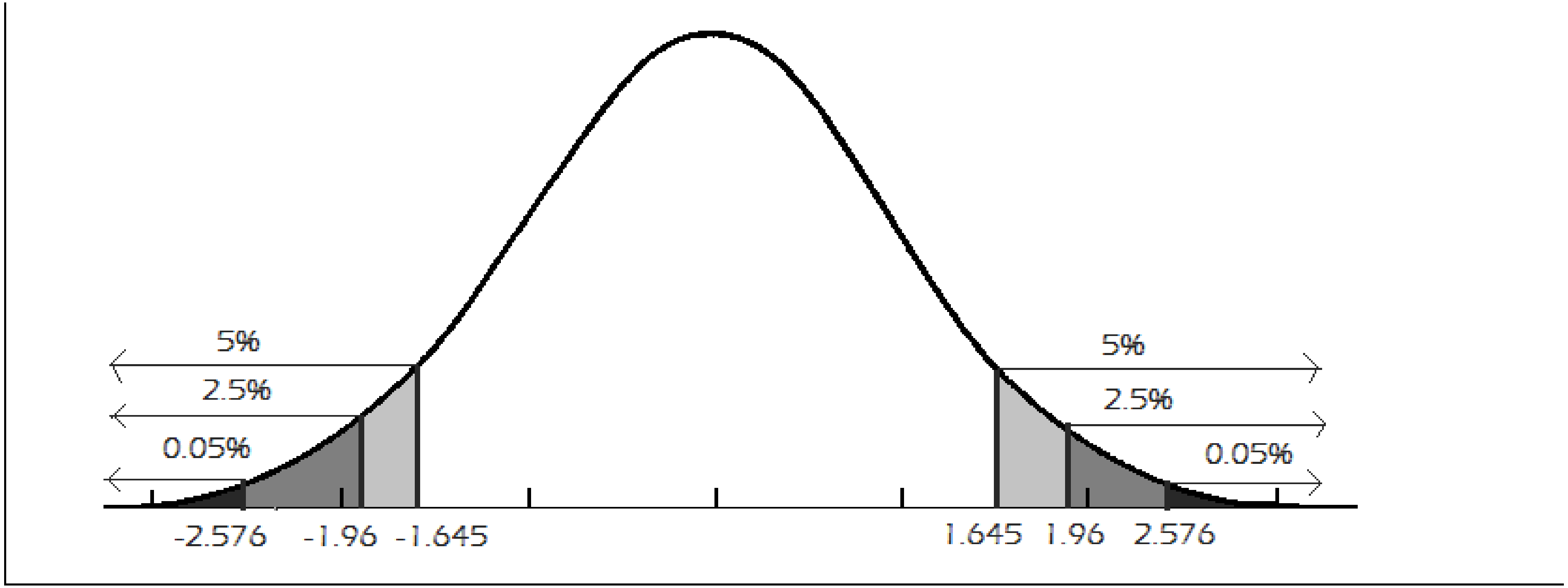

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0.95	|Test-stat|<1.960	P-value<.05
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Example: Jar Jar Binks

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- 340 randomly selected people were asked whether or not they liked Jar Jar Binks
- 23 people said that they did like
- At the 0.05 level of significance, or 95% confidence, is there evidence that less than 10% of all people like Jar Jar Binks?
- $\hat{p} = \frac{23}{340} = 0.068$

Example: Step 1

- State the Hypothesis: we are interested in whether or not **less than ten percent** of all people like Jar Jar Binks
  - $H_o: p \geq 0.10$
  - $H_a: p < 0.10$

Example: Step 2

- Check Assumptions
  - The variable is categorical
    - They like Jar Jar Binks or they don't like
  - The data was collected randomly
  - $np_o = 340 * (0.1) = 34 \geq 15$
  - $n(1 - p_o) = 340 * (0.9) = 306 \geq 15$

Example: Step 3

- Calculate the test statistic:

$$z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o(1-p_o)}{n}}} = \frac{(0.068 - 0.1)}{\sqrt{\frac{0.1(1-0.1)}{340}}} = -1.99$$

Example: Step 4

- Determine P-value for left tailed test
  - From the table $p - value = P(Z < z^*)$

$$P(Z < -1.99) = .0233$$

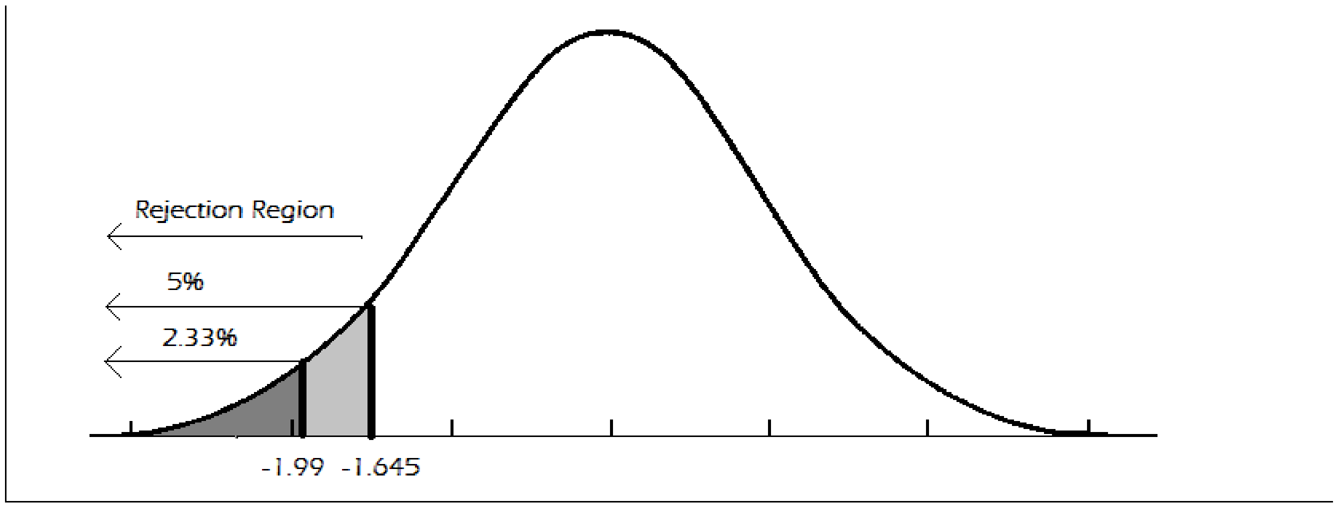
Example: Step 5

- State Conclusion
  - Since $0.0233 < 0.05$ we reject H_o

  At the 0.05 level of significance, or 95% confidence level, there is sufficient evidence that fewer than 10% of all people like Jar Jar Binks.

Example: Step 5 with Picture

- State Conclusion
  - Anything with a p-value<0.05 or a z-value<1.645 will be in the rejection region
  - Since p-value=$0.0233 < \alpha = 1 - 0.95 = 0.05$ we reject H_o



Example: Step 5 with different α

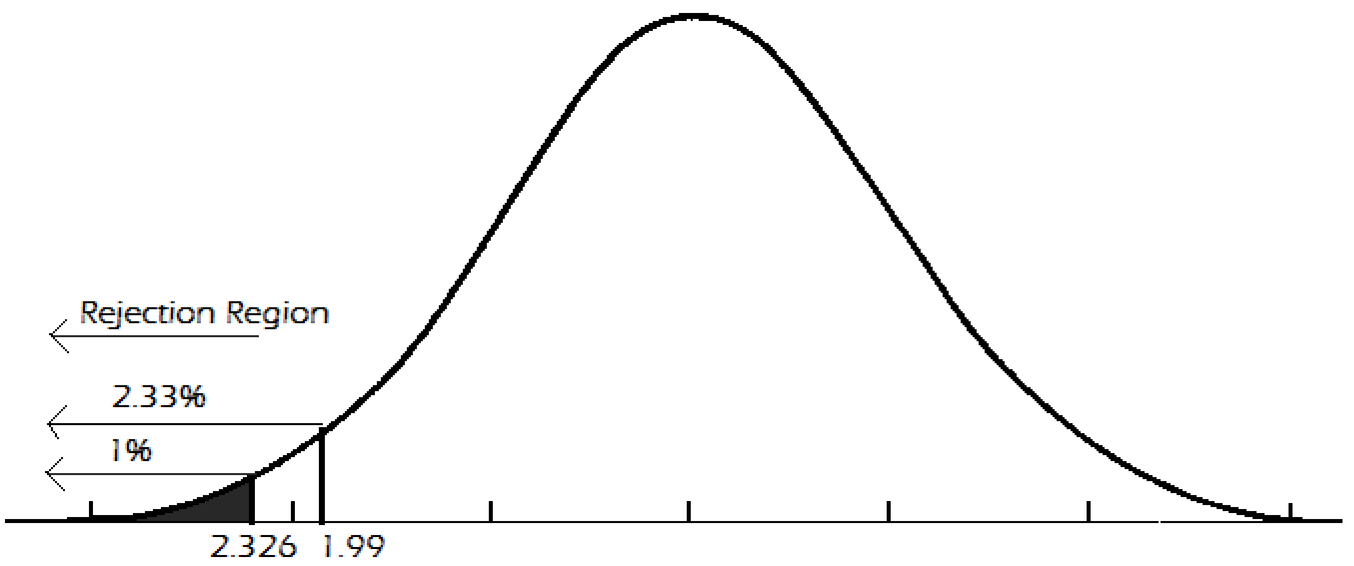
- **Note:** this would not be the case if we tested at the .01 level of significance, or 99% confidence level.

- State Conclusion
  - Since $0.0233 > 1 - 0.99 = 0.01$ we fail to reject H_o

  At the 0.01 level of significance, or 99% confidence level, there is not sufficient evidence that fewer than 10% of all people like Jar Jar Binks

Example: Step 5 with different α

- State Conclusion
  - Anything with a p-value<0.01 or a z-value<2.326 will be in the rejection region
  - Since p-value=$.0233 < \alpha = 1 - .99 = .01$ we fail to reject H_o



Example: Hogwarts

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- 1002 randomly selected students were asked whether or not they would go to Hogwarts if admitted
- 701 said they would
- At the 0.05 level of significance (95% confidence) is there evidence that the proportion of students who say they would go to Hogwarts **differs from 0.69**
- $\hat{p} = \frac{701}{1002} = 0.6996$

Example: Step 1

- State the Hypothesis: we are interested in whether or not the population proportion of people that would go to Hogwarts if accepted is **different from 0.69**
  - $H_o\colon p = 0.69$
  - $H_a\colon p \neq 0.69$

Example: Step 2

- Check Assumptions
  - The variable is categorical
    - They either would go or they wouldn't
    - The data was collected randomly
  - $np_o = 1002 * (0.69) = 691.38 \geq 15$
  - $n(1 - p_o) = 1002 * (0.31) = 310.62 \geq 15$
  - So, it is safe to assume the distribution of p_o has a bell shaped distribution

Example: Step 3

- Calculate the test statistic:

$$z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o(1 - p_o)}{n}}} = \frac{(0.6996 - 0.69)}{\sqrt{\frac{0.69(1 - 0.69)}{1002}}} = 0.6571 \approx 0.66$$

Example: Step 4

- Determine P-value for left tailed test
  - From the table $p - value = 2 * P(Z < -|z^*|)$

$$2 * P(Z < -|0.66|) = 2 * P(Z < -0.66) = 2 * 0.2545$$
$$= 0.5090$$

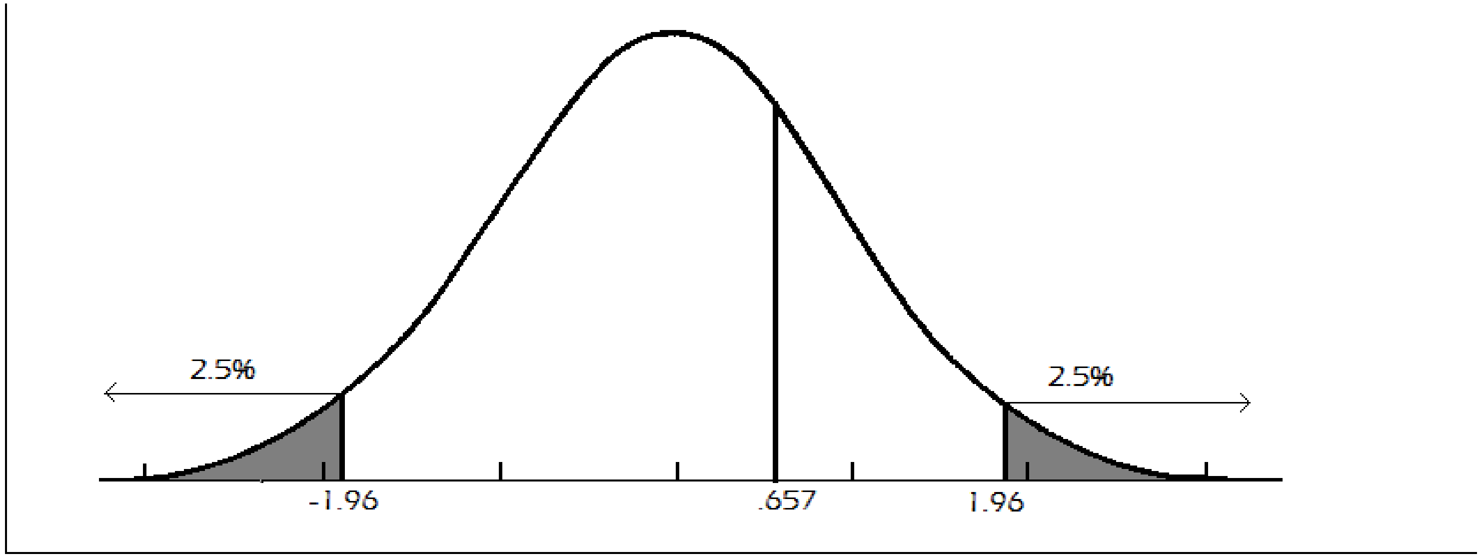
Example: Step 5

- State Conclusion
  - Since $0.5090 > 0.05$ we fail to reject H_o

  At the 0.05 level of significance, or 95% confidence level, there is not sufficient evidence that the proportion of people that would go to Hogwarts if accepted differs from 0.69

Example: Step 5 with Picture

- State Conclusion
  - Anything with a p-value<0.025 or a z-value<-1.96 or z-value>1.96 will be in the rejection region
  - Since p-value=$0.5090 < \alpha = 1 - 0.95 = 0.05$ we fail to reject H_o



Example: MLB

- Do you remember the sad story in chapter 8 about Jerry teases your boss, Tom, and you got fired?
- A random sample of MLB home games showed that the home teams won 1335 of 2429 games.
- At the 0.01 level of significance (99% confidence) is there evidence that there is a home field advantage?
- $\hat{p} = \frac{1335}{2429} = .5496$

Example: Step 1

- State the Hypotheses: we are interested in whether or not there was a home field advantage, which is whether or not the population proportion of home games won by the home team is **greater than .50**

  - $H_o: p \leq 0.5$
  - $H_a: p > 0.5$

Example: Step 2

- Check Assumptions
  - The variable is categorical
    - Either the home team won or they didn't
  - The data was collected randomly
  - $np_o = 2429 * (0.5) = 1214.5 \geq 15$
  - $n(1 - p_o) = 2429 * (0.5) = 1214.5 \geq 15$
  - So, it is safe to assume the distribution of p_o has a bell shaped distribution

Example: Step 3

- Calculate the test statistic:

$$z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o(1 - p_o)}{n}}} = \frac{(0.5496 - 0.5)}{\sqrt{\frac{0.5(1 - 0.5)}{2429}}} = 4.89$$

Example: Step 4

- Determine P-value
  - From the table $p - value = P(Z > z*)$

$$p - value = P(Z > 4.89) = 1 - P(Z < 4.89) \approx 1 - 1 = 0$$

Example: Step 5

- State Conclusion
  - Since $0 \ < 0.01$ we reject H_o

  At the 0.01 level of significance, or 99% confidence level, there is sufficient evidence to suggest that there is a home field advantage

Confidence Interval VS. Hypothesis Testing

- In Chapter 8, we have 99% confidence interval (0.523, 0.575)
- In chapter 9, we make a rejection of null with 99% confidence when asking hypothesis
  - $H_o: p \leq 0.5$
  - $H_a: p > 0.5$
- Can you see the link between the confidence interval and the hypothesis testing?