

**HW 5.1. Inferences of  $\mu_1 - \mu_2$  with known  $\sigma_1$  and  $\sigma_2$ .** The concentration of active ingredient in a liquid laundry detergent is thought to be affected by the type of catalyst used in the process. The concentration is known to have normal distribution with the standard deviation 3 grams per liter regardless of the catalyst type. Then observations on concentration are taken with each catalyst, and the data follow:

Catalyst 1: 57.9, 66.2, 65.4, 65.4, 65.2, 62.6, 67.6, 63.7, 67.2, 71.0.

Catalyst 2: 66.4, 71.7, 70.3, 69.3, 64.8, 69.6, 68.6, 69.4, 65.3, 68.8.

- (a) Find a 95% confidence interval on the difference in mean active concentrations for the two catalysts.
- (b) With significant level  $\alpha = 0.05$ , is there any evidence to indicate that the mean active concentrations depend on the choice of catalyst (i.e., they are different or not)? What is your conclusion?

**HW 5.2. Inferences of  $\mu_1 - \mu_2$  with  $\sigma_1 = \sigma_2$ .** An article in *Nature* (2003, Vol. 48, p. 1013) described an experiment in which subjects consumed different types of chocolate to determine the effect of eating chocolate on a measure of cardiovascular health. We will consider the results for only dark chocolate and milk chocolate. In the experiment, 12 subjects consumed 100 grams of dark chocolate and 200 grams of milk chocolate, one type of chocolate per day, and after one hour, the total antioxidant capacity of their blood plasma was measured in an assay. The subjects consisted of seven women and five men with an average age range of  $32.2 \pm 1$  years, an average weight of  $65.8 \pm 3.1$  kg, and average body mass index of  $21.9 \pm 0.4$  kg/m<sup>2</sup>. Data similar to that reported in the article follows.

Dark Chocolate: 118.8, 122.6, 115.6, 113.6, 119.5, 115.9, 115.8, 115.1, 116.9, 115.4, 115.6, 107.9.

Milk Chocolate: 102.1, 105.8, 99.6, 102.7, 98.8, 100.9, 102.8, 98.7, 94.7, 97.8, 99.7, 98.6.

Let  $\mu_1$  be the mean blood plasma antioxidant capacity resulting from eating dark chocolate and  $\mu_2$  be the mean blood plasma antioxidant capacity resulting from eating milk chocolate.

- (a) According to the box plots in Figure 1 or sample standard deviations, do you believe that the population standard deviations are equal? Why?
- (b) Calculate a two-sided 95% confidence interval on the mean difference  $\mu_1 - \mu_2$  of blood plasma antioxidant capacities.
- (c) Is there evidence to support the claim that consuming dark chocolate produces a higher mean level of total blood plasma antioxidant capacity than consuming milk chocolate? Use significant level  $\alpha = 0.05$ . What is your conclusion?
- (d) Do normal qq plots of blood plasma antioxidant capacity in Figure 2 indicate any violations of the assumptions for the tests and confidence interval that you performed?

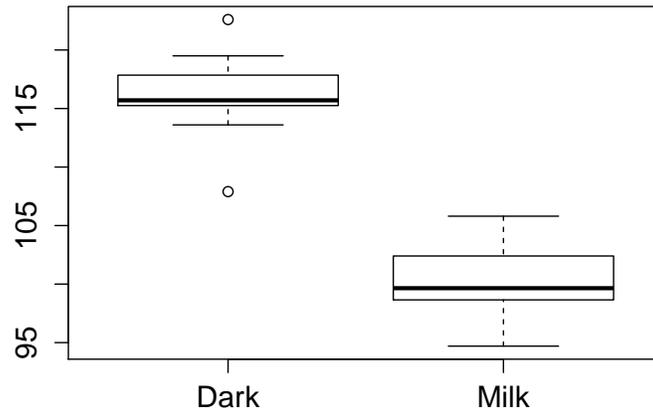


Figure 1: Boxplots for Chocolate data.

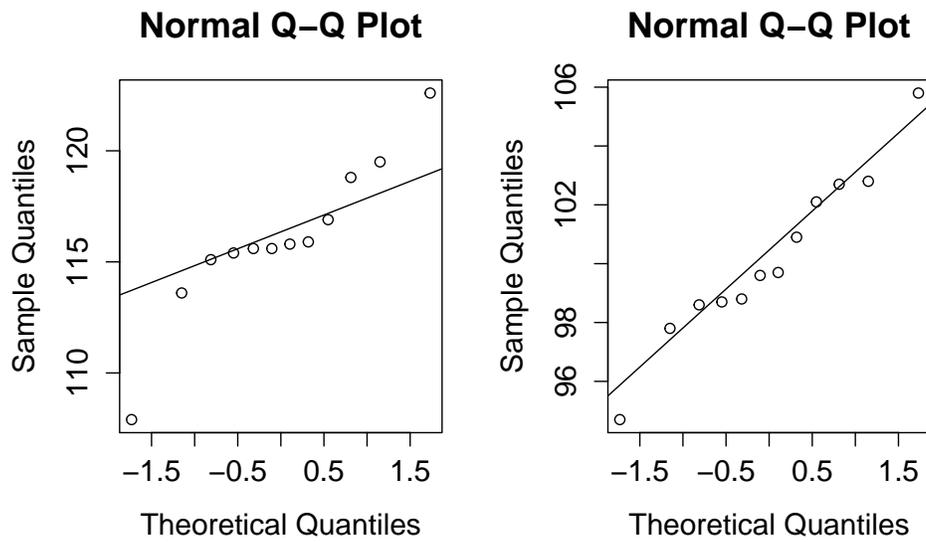


Figure 2: Normal quantile-quantile (qq) plots for Chocolate data.

**HW 5.3. Inferences of  $\mu_1 - \mu_2$  with  $\sigma_1 \neq \sigma_2$ .** An article in *Polymer Degradation and Stability* (2006, Vol. 91) presented data from a nine-year aging study on S537 foam. Foam samples were compressed to 50% of their original thickness and stored at different temperatures for nine years. At the start of the experiment as well as during each year, sample thickness was measured, and the thicknesses of the eight samples at each storage condition were recorded. The data of two storage conditions follow:

50°C: 0.047, 0.060, 0.061, 0.064, 0.080, 0.090, 0.118, 0.165, 0.183.

60°C: 0.062, 0.105, 0.118, 0.137, 0.153, 0.197, 0.210, 0.250, 0.375.

- (a) According to the box plots in Figure 3 or standard deviations, what do you believe:  $\sigma_1 = \sigma_2$  or  $\sigma_1 \neq \sigma_2$ ? Why?
- (b) Find a 95% confidence interval for the difference in the mean compression for the two temperatures.
- (c) Is there evidence to support the claim that mean compression increases with the temperature at the storage condition?
- (d) Do normal qq plots of compression in Figure 4 indicate any violations of the assumptions for the tests and confidence interval that you performed?

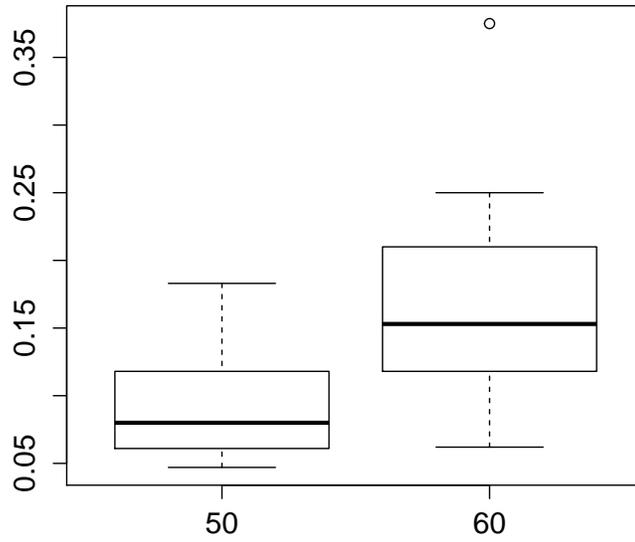


Figure 3: Boxplots for Foam data.

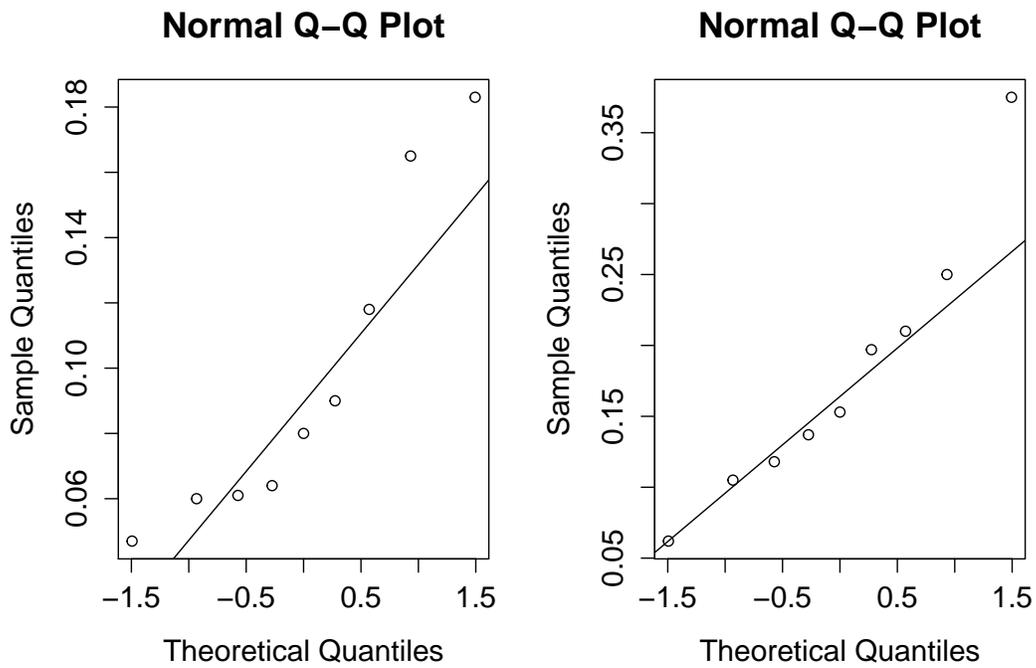


Figure 4: Normal quantile-quantile (qq) plots for Foam data.

**HW 5.4. Inference of  $p_1 - p_2$ .** An article in *Knee Surgery, sports traumatology, Arthroscopy* (2005, Vol. 13, pp. 273-279) considered arthroscopic meniscal repair with an absorbable screw. Results showed that for tears greater than 25 millimeters, 14 of 18 (78%) repairs were successful, but for shorter tears, 22 of 30 (73%) repairs were successful.

- (a) Calculate a one-sided 95% confidence bound on the difference in proportions.
- (b) Is there evidence that the success rate is greater for longer tears? Use significant level  $\alpha = 0.05$ .  
What is your conclusion?

**HW 5.5. Inferences of  $\sigma_1^2/\sigma_2^2$ .** The thickness of a plastic film (in mils) on a substrate material is thought to be influenced by the temperature at which the coating is applied. In a completely randomized experiment, 11 substrates are coated at 125°F, resulting in a sample mean coating thickness of sample mean  $\bar{x}_1 = 103.5$  and a sample standard deviation of  $s_1 = 10.2$ . Another 13 substrates are coated at 150°F for which sample mean  $\bar{x}_2 = 99.7$  and sample standard deviation  $s_2 = 20.1$  are observed. It was originally suspected that raising the process temperature would reduce mean coating thickness.

(a) Construct a 95% confidence interval for  $\sigma_1^2/\sigma_2^2$ .

(b) Test  $H_0 : \sigma_1 = \sigma_2$  against  $H_a : \sigma_1 > \sigma_2$  using  $\alpha = 0.05$ . What is your conclusion?

**HW 5.6. Inferences of  $\mu_1 - \mu_2$  in matched pairs design.** The manager of a fleet of automobiles is testing two brands of radial tires and assigns one tire of each brand at random to the two rear wheels of eight cars and runs the cars until the tires wear out. The data (in kilometers) follow:

Car	Brand 1	Brand 2
1	36,925	34,318
2	45,300	42,280
3	36,240	35,500
4	32,100	31,950
5	37,210	38,015
6	48,360	47,800
7	38,200	37,810
8	33,500	33,215

- (a) Find a 95% confidence interval on the difference in mean life.
- (b) Which brand would you prefer? Test  $H_0 : \mu_D = 0$  against  $\mu \neq 0$  using  $\alpha = 0.05$ . What is your conclusion?

**HW 5.7.** The compressive strength of concrete is being studied, and four different mixing techniques are being investigated. The following data have been collected. Test the hypothesis that mixing techniques affect the strength of the concrete. Use  $\alpha = 0.05$ .

Mixing Technique	Compressive Strength (psi)			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

**HW 5.8.** An electronics engineer is interested in the effect on tube conductivity of five different types of coating for cathode ray tubes in a telecommunications system display device. The following conductivity data are obtained. Is there any difference in conductivity due to coating type? Use  $\alpha = 0.01$ .

Coating Type	Conductivity			
1	143	141	150	146
2	152	149	137	143
3	134	133	132	127
4	129	127	132	129
5	147	148	144	142