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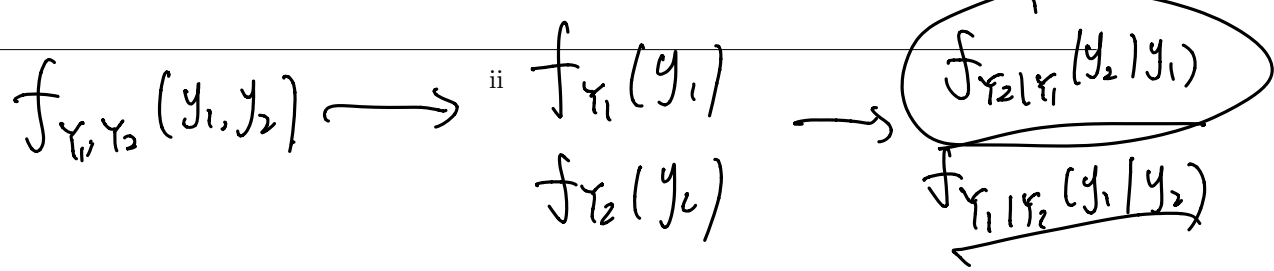
pmf  
cdf  
 $\mu$   
 $\sigma^2$   
Prob  
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$$\Gamma(\alpha) \beta^\alpha = \int_0^{\infty} y^{\alpha-1} e^{-y/\beta} dy$$

$$\frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha+\beta)} = \int_0^1 y^{\alpha-1} (1-y)^{\beta-1} dy$$

pdf  
cdf  
 $\mu$   
 $\sigma^2$   
Mgf  
Prob  
Poisson Process



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$Cov(Y_1, Y_2) = E(Y_1 Y_2) - E(Y_1) \cdot E(Y_2)$

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→  $E(Y_2 | Y_1 = y_1) = \int y_2 f_{Y_2 | Y_1 = y_1}(y_2 | y_1) dy_2$

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