HW 1-2 (Due Aug. 25, 2016) **Name:**

Problem 1. If A and B are two sets, draw Venn diagrams to verify the followings:

1. $A = (A \cap B) \cup (A \cap \bar{B})$
2. if $B \subset A$ then $A = B \cup (A \cap \bar{B})$

Use the identities $A = A \cap S$ and $S = B \cup \bar{B}$ and a distributive law to prove that (mathematically, not graphically)

1. $A = (A \cap B) \cup (A \cap \bar{B})$
2. if $B \subset A$ then $A = B \cup (A \cap \bar{B})$
3. Further, show that $(A \cap B)$ and $(A \cap \bar{B})$ are mutually exclusive and therefore that A is the union of two mutually exclusive sets, $(A \cap B)$ and $(A \cap \bar{B})$.
4. Also show that B and $(A \cap \bar{B})$ are mutually exclusive and if $B \subset A$, A is the union of two mutually exclusive sets, B and $(A \cap \bar{B})$.

Problem 2. Suppose two dice are tossed and the numbers on the upper faces are observed. Let S denote the set of all possible pairs that can be observed. For example, let $(2, 3)$ denote that a 2 was observed on the first die and a 3 on the second.

1. Define the following subsets of S: A: The number on the second die is even. B: The sume of the two numbers is even. C: At least one number in the pair is odd. Using equally likely rule to calculate $P(A)$, $P(B)$ and $P(C)$.

2. List the points in A, $\bar{C}$, $A \cap B$, $A \cap \bar{B}$, $\bar{A} \cup B$ and $\bar{A} \cap C$.

Problem 3. Suppose two balanced coins are tossed and the upper faces are observed.

1. List the sample points for this experiment
2. Assign a reasonable probability to each sample point. (Are the sample points equally likely?)
3. Let A denote the event that exactly one head is observed and B the event that at least one head is observed. List the sample points in both A and B.
4. From your answer to part 3., find $P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$ and $P(\bar{A} \cup B)$.