HW 10-1 (Due Nov. 22, 2016) **Name:**

Print then work on it directly. Staple HW 10-1 and 10-2 together.

Problem 1

5.26 In Exercise 5.8, we derived the fact that

$$f(y_1, y_2) = \begin{cases} 4y_1y_2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint probability density function. Find

a the marginal density functions for Y_1 and Y_2.

b $P(Y_1 \le 1/2 | Y_2 \ge 3/4)$.

c the conditional density function of Y_1 given $Y_2 = y_2$.

d the conditional density function of Y_2 given $Y_1 = y_1$.

e $P(Y_1 \le 3/4 | Y_2 = 1/2)$.

Problem 2

5.27 In Exercise 5.9, we determined that

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \le y_1 \le y_2 \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint probability density function. Find

a the marginal density functions for Y_1 and Y_2.

b $P(Y_2 \le 1/2 | Y_1 \le 3/4)$.

c the conditional density function of Y_1 given $Y_2 = y_2$.

d the conditional density function of Y_2 given $Y_1 = y_1$.

e $P(Y_2 \ge 3/4 | Y_1 = 1/2)$.

Problem 3

5.31 In Exercise 5.13, the joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \begin{cases} 30y_1y_2^2, & y_1 - 1 \le y_2 \le 1 - y_1, 0 \le y_1 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Show that the marginal density of Y_1 is a beta density with $\alpha = 2$ and $\beta = 4$.

b Derive the marginal density of Y_2.

c Derive the conditional density of Y_2 given $Y_1 = y_1$.

d Find $P(Y_2 > 0|Y_1 = .75)$.

Problem 4

5.32 Suppose that the random variables Y_1 and Y_2 have joint probability density function, $f(y_1, y_2)$, given by (see Exercise 5.14)

$$f(y_1, y_2) = \begin{cases} 6y_1^2 y_2, & 0 \le y_1 \le y_2,\ y_1 + y_2 \le 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Show that the marginal density of Y_1 is a beta density with $\alpha = 3$ and $\beta = 2$.

b Derive the marginal density of Y_2.

c Derive the conditional density of Y_2 given $Y_1 = y_1$.

d Find $P(Y_2 < 1.1 | Y_1 = .60)$.

Problem 5

5.61 In Exercise 5.18, Y_1 and Y_2 denoted the lengths of life, in hundreds of hours, for components of types I and II, respectively, in an electronic system. The joint density of Y_1 and Y_2 is

$$f(y_1, y_2) = \begin{cases} (1/8)y_1 e^{-(y_1+y_2)/2}, & y_1 > 0, y_2 > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Are Y_1 and Y_2 independent?

Why?

Problem 6

5.63 Let Y_1 and Y_2 be independent exponentially distributed random variables, each with mean 1. Find $P(Y_1 > Y_2 \mid Y_1 < 2Y_2)$.