HW 10-2 (Due Nov. 22, 2016) **Name:**

Print then work on it directly. Staple HW 10-1 and 10-2 together.

Problem 1 Suppose that Y_1 and Y_2 are random variables (discrete or continuous). Prove the following results:

(a) $Cov(Y_1, Y_2) = Cov(Y_2, Y_1)$.

(b) $Cov(Y_1, Y_1) = V(Y_1)$

(c) $Cov(a + bY_1, c + dY_2) = bdCov(Y_1, Y_2)$, for any constants a, b, c, and d.

Problem 2 Check Example 5.17 on Page 127 of the lecture notes. Find $E(U_2)$ and $V(U_2)$. And then compute $Cov(U_1, U_2)$.

Problem 3

5.77 In Exercise 5.9, we determined that

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \le y_1 \le y_2 \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint probability density function. Find

a $E(Y_1)$ and $E(Y_2)$.

b $V(Y_1)$ and $V(Y_2)$.

c $E(Y_1 - 3Y_2)$.

Problem 4

5.78 In Exercise 5.10, we proved that

$$f(y_1, y_2) = \begin{cases} 1, & 0 \le y_1 \le 2, 0 \le y_2 \le 1, 2y_2 \le y_1, \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint probability density function for Y_1, the amount of pollutant per sample collected above the stack without the cleaning device, and Y_2, the amount collected above the stack with the cleaner.

a Find $E(Y_1)$ and $E(Y_2)$.

b Find $V(Y_1)$ and $V(Y_2)$.

c The random variable $Y_1 - Y_2$ represents the amount by which the weight of pollutant can be reduced by using the cleaning device. Find $E(Y_1 - Y_2)$.

d Find $V(Y_1 - Y_2)$. Within what limits would you expect $Y_1 - Y_2$ to fall?

Problem 5

5.92 In Exercise 5.9, we determined that

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2), & 0 \le y_1 \le y_2 \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

is a valid joint probability density function. Find $\text{Cov}(Y_1, Y_2)$. Are Y_1 and Y_2 independent?

Problem 6

5.107 In Exercise 5.12, we were given the following joint probability density function for the random variables Y_1 and Y_2, which were the proportions of two components in a sample from a mixture of insecticide:

$$f(y_1, y_2) = \begin{cases} 2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1, 0 \le y_1 + y_2 \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

For the two chemicals under consideration, an important quantity is the total proportion $Y_1 + Y_2$ found in any sample. Find $E(Y_1 + Y_2)$ and $V(Y_1 + Y_2)$.