HW 6-2 (Due Oct. 18, 2016) **Name:**

Print then work on it directly. Staple HW 6-1 and 6-2 together.

Problem 1 Prove that:

4.1 Let Y be a random variable with $p(y)$ given in the table below.

y	1	2	3	4
$p(y)$	.4	.3	.2	.1

a Give the distribution function, $F(y)$. Be sure to specify the value of $F(y)$ for all y, $-\infty < y < \infty$.

b Sketch the distribution function given in part (a).

Problem 2

4.2 A box contains five keys, only one of which will open a lock. Keys are randomly selected and tried, one at a time, until the lock is opened (keys that do not work are discarded before another is tried). Let Y be the number of the trial on which the lock is opened.

a Find the probability function for Y.

b Give the corresponding distribution function.

c What is $P(Y < 3)$? $P(Y \leq 3)$? $P(Y = 3)$?

d If Y is a continuous random variable, we argued that, for all $-\infty < a < \infty$, $P(Y = a) = 0$. Do any of your answers in part (c) contradict this claim? Why?

Problem 3

4.5 Suppose that Y is a random variable that takes on only integer values $1, 2, \ldots$ and has distribution function $F(y)$. Show that the probability function $p(y) = P(Y = y)$ is given by

$$p(y) = \begin{cases} F(1), & y = 1, \\ F(y) - F(y-1), & y = 2, 3, \ldots. \end{cases}$$

Problem 4

4.8 Suppose that Y has density function

$$f(y) = \begin{cases} ky(1-y), & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the value of k that makes $f(y)$ a probability density function.

b Find $P(.4 \le Y \le 1)$.

c Find $P(.4 \le Y < 1)$.

d Find $P(Y \le .4 | Y \le .8)$.

e Find $P(Y < .4 | Y < .8)$.

Problem 5

4.11 Suppose that Y possesses the density function

$$f(y) = \begin{cases} cy, & 0 \leq y \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the value of c that makes $f(y)$ a probability density function.

b Find $F(y)$.

c Graph $f(y)$ and $F(y)$.

d Use $F(y)$ to find $P(1 \leq Y \leq 2)$.

e Use $f(y)$ and geometry to find $P(1 \leq Y \leq 2)$.

Problem 6

4.14 A gas station operates two pumps, each of which can pump up to 10,000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable Y (measured in 10,000 gallons) with a probability density function given by

$$f(y) = \begin{cases} y, & 0 < y < 1, \\ 2 - y, & 1 \leq y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Graph $f(y)$.

b Find $F(y)$ and graph it.

c Find the probability that the station will pump between 8000 and 12,000 gallons in a particular month.

d Given that the station pumped more than 10,000 gallons in a particular month, find the probability that the station pumped more than 15,000 gallons during the month.