

HW 7-1 (Due Oct. 25, 2016)

Name:

Print then work on it directly. Staple HW 7-1 and 7-2 together.

Problem 1 Prove that:

- 4.12** The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function given by

$$F(y) = \begin{cases} 0, & y < 0, \\ 1 - e^{-y^2}, & y \geq 0. \end{cases}$$

- a** Show that $F(y)$ has the properties of a distribution function.
- b** Find the .30-quantile, $\phi_{.30}$, of Y .
- c** Find $f(y)$.
- d** Find the probability that the transistor operates for at least 200 hours.
- e** Find $P(Y > 100|Y \leq 200)$.

Problem 2

- 4.15** As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \begin{cases} \frac{b}{y^2}, & y \geq b, \\ 0, & \text{elsewhere,} \end{cases}$$

where b is the minimum possible time needed to traverse the maze.

- a** Show that $f(y)$ has the properties of a density function.
- b** Find $F(y)$.
- c** Find $P(Y > b + c)$ for a positive constant c .
- d** If c and d are both positive constants such that $d > c$, find $P(Y > b + d | Y > b + c)$.

Problem 3

4.18 Let Y have the density function given by

$$f(y) = \begin{cases} .2, & -1 < y \leq 0, \\ .2 + cy, & 0 < y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a** Find c .
- b** Find $F(y)$.
- c** Graph $f(y)$ and $F(y)$.
- d** Use $F(y)$ in part (b) to find $F(-1)$, $F(0)$, and $F(1)$.
- e** Find $P(0 \leq Y \leq .5)$.
- f** Find $P(Y > .5 | Y > .1)$.

Problem 4 Suppose Y has the following density function,

$$f(y) = \begin{cases} (3/2)y^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere,} \end{cases}$$

- (a) Find $E(Y)$ and $V(Y)$.
- (b) Find the moment generating function, $m_Y(t)$, of Y .
- (c) Use $m_Y(t)$ to derive $E(Y)$ and $V(Y)$. Check them with the ones of part (a).

Problem 5 Suppose Y has the following cdf,

$$F(y) = \begin{cases} 0, & y \leq 0, \\ \frac{y}{8}, & 0 < y < 2, \\ \frac{y^2}{16}, & 2 \leq y < 4, \\ 1, & y \geq 4, \end{cases}$$

find the mean and variance of Y .

Problem 6

- 4.31** The pH of water samples from a specific lake is a random variable Y with probability density function given by

$$f(y) = \begin{cases} (3/8)(7 - y)^2, & 5 \leq y \leq 7, \\ 0, & \text{elsewhere.} \end{cases}$$

- a** Find $E(Y)$ and $V(Y)$.
- b** Find an interval shorter than $(5, 7)$ in which at least three-fourths of the pH measurements must lie.
- c** Would you expect to see a pH measurement below 5.5 very often? Why?