**HW 7-2 (Due Oct. 25, 2016)** **Name:**

Print then work on it directly. Staple HW 7-1 and 7-2 together.

**Problem 1** Prove that:

**4.43** A circle of radius $r$ has area $A = \pi r^2$. If a random circle has a radius that is uniformly distributed on the interval $(0, 1)$, what are the mean and variance of the area of the circle?

**Problem 2**

**4.47** The failure of a circuit board interrupts work that utilizes a computing system until a new board is delivered. The delivery time, $Y$, is uniformly distributed on the interval one to five days. The cost of a board failure and interruption includes the fixed cost $c_0$ of a new board and a cost that increases proportionally to $Y^2$. If $C$ is the cost incurred, $C = c_0 + c_1 Y^2$.

**a** Find the probability that the delivery time exceeds two days.

**b** In terms of $c_0$ and $c_1$, find the expected cost associated with a single failed circuit board.

**Problem 3**

**4.58** Use Table 4, Appendix 3, to find the following probabilities for a standard normal random variable Z:

**a** $P(0 \leq Z \leq 1.2)$
**b** $P(-.9 \leq Z \leq 0)$
**c** $P(.3 \leq Z \leq 1.56)$

Ignore the words "Use Table 4, Appendix 3." Just find the above probabilities.

**4.59** If $Z$ is a standard normal random variable, find the value $z_0$ such that

**a** $P(Z > z_0) = .5$.
**b** $P(Z < z_0) = .8643$.
**c** $P(-z_0 < Z < z_0) = .90$.
**d** $P(-z_0 < Z < z_0) = .99$.

**4.62** If $Z$ is a standard normal random variable, what is

**a** $P(Z^2 < 1)$?
**b** $P(Z^2 < 3.84146)$?

**Problem 4**

**4.72** One method of arriving at economic forecasts is to use a consensus approach. A forecast is obtained from each of a large number of analysts; the average of these individual forecasts is the consensus forecast. Suppose that the individual 1996 January prime interest–rate forecasts of all economic analysts are approximately normally distributed with mean 7% and standard deviation 2.6%. If a single analyst is randomly selected from among this group, what is the probability that the analyst's forecast of the prime interest rate will

**a** exceed 11%?

**b** be less than 9%?

**Problem 5**

**4.74** Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

- **a** What is the probability that a person taking the examination scores higher than 72?
- **b** Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?
- **c** What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?
- **d** Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25%?
- **e** **Applet Exercise** Answer parts (a)–(d), using the applet *Normal Tail Areas and Quantiles*.
- **f** If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84?

Ignore part e.

**Problem 6**

**4.80** Assume that $Y$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. After observing a value of $Y$, a mathematician constructs a rectangle with length $L = |Y|$ and width $W = 3|Y|$. Let $A$ denote the area of the resulting rectangle. What is $E(A)$?