

**HW 8-2 (Due Nov. 1, 2016)**

**Name:**

Print then work on it directly. Staple HW 8-1 and 8-2 together.

**Problem 1**

**4.96** Suppose that a random variable  $Y$  has a probability density function given by

$$f(y) = \begin{cases} ky^3e^{-y/2}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- a** Find the value of  $k$  that makes  $f(y)$  a density function.
- b** Does  $Y$  have a  $\chi^2$  distribution? If so, how many degrees of freedom?
- c** What are the mean and standard deviation of  $Y$ ?

**Problem 2**

- 4.109** The weekly amount of downtime  $Y$  (in hours) for an industrial machine has approximately a gamma distribution with  $\alpha = 3$  and  $\beta = 2$ . The loss  $L$  (in dollars) to the industrial operation as a result of this downtime is given by  $L = 30Y + 2Y^2$ . Find the expected value and variance of  $L$ .

**Problem 3**

**4.110** If  $Y$  has a probability density function given by

$$f(y) = \begin{cases} 4y^2e^{-2y}, & y > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

obtain  $E(Y)$  and  $V(Y)$  by inspection.

Find the probability  $P(Y > 1)$ .

**Problem 4**

**4.123** The relative humidity  $Y$ , when measured at a location, has a probability density function given by

$$f(y) = \begin{cases} ky^3(1-y)^2, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a** Find the value of  $k$  that makes  $f(y)$  a density function.
- b.** Find  $P(0.2 < Y < 0.8)$ .

**Problem 5**

**4.126** The weekly repair cost  $Y$  for a machine has a probability density function given by

$$f(y) = \begin{cases} 3(1 - y)^2, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

with measurements in hundreds of dollars. How much money should be budgeted each week for repair costs so that the actual cost will exceed the budgeted amount only 10% of the time?

**Problem 6**

**4.128** Suppose that a random variable  $Y$  has a probability density function given by

$$f(y) = \begin{cases} 6y(1 - y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- a** Find  $F(y)$ .
- b** Graph  $F(y)$  and  $f(y)$ .
- c** Find  $P(.5 \leq Y \leq .8)$ .

**Problem 7**

- 4.129** During an eight-hour shift, the proportion of time  $Y$  that a sheet-metal stamping machine is down for maintenance or repairs has a beta distribution with  $\alpha = 1$  and  $\beta = 2$ . That is,

$$f(y) = \begin{cases} 2(1 - y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The cost (in hundreds of dollars) of this downtime, due to lost production and cost of maintenance and repair, is given by  $C = 10 + 20Y + 4Y^2$ . Find the mean and variance of  $C$ .