Notes_2_1_Introduction

Thursday, August 18, 2016 12:37 PM

Notes_2_1_ Introducti...

2 Probability

Complementary reading: Chapter 2 (WMS).

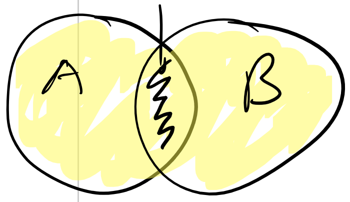
2.1 Introduction

TERMINOLOGY: The text defines **probability** as a measure of one's belief in the occurrence of a future (random) event. Probability is also known as "the mathematics of uncertainty."

Area :

Intersection of A and B

$A \cap B$



$A \cup B$

↳union of A, B

$$\text{Area}(A \cup B) = \text{Area}(A) + \text{Area}(B) - \text{Area}(A \cap B)$$

REAL LIFE EVENTS: Here are some events we may wish to assign probabilities to:

- tomorrow's temperature exceeding 80 degrees
- getting a flat tire on my way home today
- a new policy holder making a claim in the next year
- the NASDAQ losing 5 percent of its value this week
- you being diagnosed with prostate/cervical cancer in the next 20 years.

ASSIGNING PROBABILITIES: How do we assign probabilities to events? There are three general approaches.

1. *Subjective approach.*
   - This approach is based on feeling and may not even be scientific.
2. *Relative frequency approach.*
   - This approach can be used when some random phenomenon is observed repeatedly under identical conditions.
3. *Axiomatic/Model-based approach.* This is the approach we will take in this course.

Ex. flip a coin.

$P(\text{Head}) = \frac{1}{2}$?

equal chance rule.

flip n times. $n(\text{head})$

$$\lim_{n \to \infty} \frac{n(\text{head})}{n} \to \frac{1}{2}$$

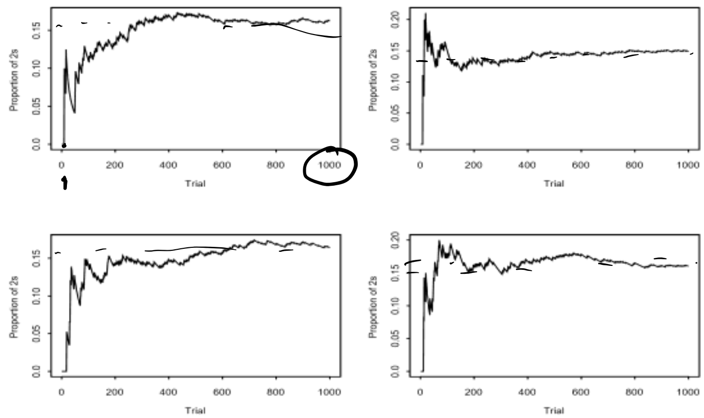


Figure 2.1: *The relative frequency of die rolls which result in a "2"; each plot represents* 1000 *simulated rolls of a fair die.*

$\frac{1}{6} \approx 0.166666$

Example 2.1. *Relative frequency approach.* Suppose that we roll a die 1000 times and record the number of times we observe a "2." Let A denote this event. The **relative frequency approach** says that

$$P(A) \approx \frac{\text{number of times } A \text{ occurs}}{\text{number of trials performed}} = \frac{n(A)}{n},$$

where $n(A)$ denotes the **frequency** of the event, and n denotes the number of trials performed. The proportion $n(A)/n$ is called the **relative frequency**. The symbol $P(A)$ is shorthand for "the probability that A occurs."

RELATIVE FREQUENCY APPROACH: Continuing with our example, suppose that $n(A) = 158$. We would then estimate $P(A)$ by $158/1000 = 0.158$. If we performed the experiment of rolling a die repeatedly, the relative frequency approach says that

$$\frac{n(A)}{n} \to P(A),$$

as $n \to \infty$. Of course, if the die is fair, then $n(A)/n \to P(A) = 1/6$. □

$P(\overbrace{\text{a male 20 years Prostate Cancer}}^{\text{Event}})$

$= P(\text{Event} \mid \underbrace{\text{an individual's information}}_{x})$

$= \underbrace{P(\text{Event} \mid x)}_{\text{model}}$

$= f(x)$

$\hat{p} = \hat{f}(x)$